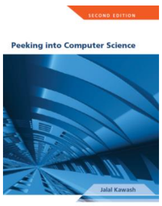
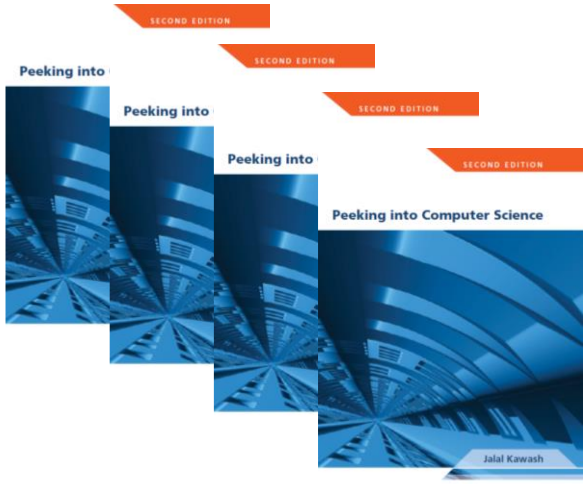
1 Logic

Peeking into Computer Science



Reading Assignment

- Mandatory: Chapter 2 – Section 2.1



Propositional Logic

By the end of this section, you will be able to:

1. Define a proposition
2. Define and combine the logic operators: AND, OR, NOT, XOR, and implies
3. Use truth tables to determine equivalence of propositions
4. Determine if a proposition is a tautology, contradiction, or contingency

Objectives

- Because students need more help with this section: There will be several extra slides and examples covered in class in addition to the material covered in the online version.

JT's Extra-Extra

- A proposition is a statement whose value is either TRUE or FALSE

- It is snowing today
- I am not older than you
- Canada is the largest country
- Canada shares a border with the US

JT's Extra: recall computers always work on a two state model (on/off, true/false, pitted/smooth etc.) which is one reason why logic is included in this course.

Propositions and Truth Values

- X > 12
- Mr. X is taller than 250cm
- X+Y = 5
- Mrs. Y weighs 50kg

- Unless we know what the values of X and Y are, these are not propositions

Not Propositions

- The number 5
- lolz!

JT's Extra: Not Propositions

- Propositions can be built from other propositions using logical operators:
  - AND, OR, NOT, and XOR (exclusive OR)

- It is raining today **AND** it is very warm
- At 12:00 today, I will be eating **OR** I will be home (inclusive OR)
- I will be either at the beach **OR** hiking (XOR)
  - JT: this is *exclusive-OR,* having one case true *excludes* the possibility of the other being true)
- I will **NOT** be home at 6

Compound propositions

- The popular usage of the logical AND applies when *ALL* conditions must be met (very stringent).
  - In terms of logic: 'AND' applies when all propositions must be true.
  - Example: I picked up all the kids today.
  - Picked up son AND picked up daughter.

JT's Extra: Logical AND

- The correct everyday usage of the logical OR applies when *ATLEAST* one condition must be met (less stringent).
  - In terms of logic: 'OR' applies when one or more propositions are true.
  - Example:
  - Using the 'Peeking' book OR using another book to supplement your learning.

JT's Extra: Logical OR

- The everyday usage of logical NOT negates (or reverses) a statement.
- In terms of logic: 'NOT' reverses a proposition (true becomes false and false becomes true).
- I am finding this class quite stimulating and exciting NOT!!!

Statement (logical condition)

Negation of the statement/condition

JT's Extra: Logical NOT

- Double negation in logic is similar to mathematics

- - 10

I am finding this class quite stimulating and exciting *NOT!!!* *NOT!!!*

JT's Extra: Double Negation

- Bachelor of Commerce (Year 1) Required Grade 12 High School Subject
  - English 30 or ELA 30-1 and
  - Pure Mathematics 30 and
  - Subject from Group A or B

Example: U of C Calendar

Eligible tuition fees

Generally, a course qualifies if it was taken at the post-secondary level or (for individuals aged 16 or over at the end of the year) it develops or improves skills in an occupation and the educational institution has been certified by Human Resources and Social Development Canada. In addition, you must have taken the course in 2007.

Example: Income Tax Guide

Line 349 - Donations and gifts

You can claim donations either you or your spouse or common-law partner made. For more information about donations and gifts, or if you donated any of the following:

- gifts of property other than cash; or
- gifts to organizations outside Canada; or
- gifts to Canada, a province, or a territory made after 1997 and agreed to in writing before February 19, 1997.

Example: Income Tax Guide

- If A is a proposition,
- then ¬A is a proposition
- that is true when A is false
- And false when A is true

- ¬A is read *NOT A*

Negation (JT 'NOT')

A	¬ A
T	F
F	T

It is raining today

It is **not** raining today

Truth Table for Negation

- If A and B are a propositions,
- then A ∧ B is a proposition
- that is true when both A and B are true
- otherwise, it is false

- A ∧ B is read *A and B*

Conjunction (JT: 'AND')

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A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

It is raining today **and** it is cold

JT: Column A **JT: Column B**

Truth Table for Conjunction

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Disjunction (JT: 'OR')

- If A and B are a propositions,
- then A $\vee$ B is a proposition
- that is false when both A and B are false
- otherwise, it is true

- A $\vee$ B is read *A or B*

Truth Table for Disjunction

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

$\underbrace{\text{It is raining today}}_{\text{JT: Column A}}$ **or** $\underbrace{\text{it is cold}}_{\text{JT: Column B}}$

- In order to be hired for this position applicants must have at least one of the following: a grade point over 3.0, at least 5 years of relevant job experience.
  - Alice: GPA 3.7, 10 years experience.
  - Bob: GPA 4.0, no work experience.
  - Charlie: GPA = 2.0, 5 years work experience.
  - Jim: GPA = 1.7, 1 year work experience.

X

√

JT's Extra: Who Gets Hired?

- The company will be downsized with the following group of people to be laid off: employees who were on the job for less than two years and earn over $50,000 per year.
  - Alice: 10 years on the job, $30,000 salary.
  - Bob: 5 years on the job, $100,000 salary.
  - Charlie: 1 year on the job, $45,000 salary.
  - Jim: 0 years on the job, $1,000,000 salary.

X

√

JT's Extra: Who Do We Let Go?

- Keen eye for fashion: true or false?
  - It is not the case that the course instructor for this CPSC 203 lecture has a keen fashion sense.

X

√

Your Ace Game Show Host Jim Tam ™

JT's Extra: Fashion Sense?

- JT's Extra: sometimes the everyday usage of 'OR' does not correspond to 'Logical OR'.

  Example: Flight attendant asks if you would like beer or wine?

- To be eligible for the job, you have to be above 21 ***or*** attending a post-secondary institution
- We'll drive you to your place: my friend ***or*** I will be driving your car

Inclusive Vs. Exclusive OR

Exclusive OR

- If A and B are a propositions,
- then A ⊕ B is a proposition
- that is true when exactly one of A and B is true
- otherwise, it is false

- A ⊕ B is read *A xor B*

Truth Table for XOR

A	B	A ⊕ B
T	T	F
T	F	T
F	T	T
F	F	F

It will be at home **or** at school

1. It will rain or it will not rain today.
2. Either "Santa's little helper"[1] or "Lucky 7" will win the race. (Assume clear cut outcome).
3. You can take it or leave it.
4. Her grades are excellent so she's either very bright or studies hard.

1) From "The Simpsons" © Fox

JT's Extra: Inclusive Or Exclusive?

- Use truth tables to prove the equivalence of the two sets of logical expressions:
  - NOT (A OR B)
  - Is equivalent to
  - NOT(A) AND NOT(B)

JT: In Class Exercise

JT's Extra: Another Equivalency

- You have just proved DeMorgan's Law
  - Example from
    - http://people.hofstra.edu/Stefan_Waner/RealWorld/logic/logic2.html

Let p: "The President is a Democrat,"

Let q: "The President is a Republican."

Then ~(p AND q): "The President is not both a Democrat and a Republican."

This is the same as saying: "Either the President is not a Democrat, or he is not a Republican, or he is neither," which is (~p) OR (~q).

XOR is Redundant

A	B	A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
T	T	F	T	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	F	F	T

A xor B = (A or B) and not (A and B)

A	B	A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
T	T	F	T	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	F	F	T

$$A \oplus B = \boxed{(A \vee B)} \wedge \boxed{\neg (A \wedge B)}$$

XOR is Redundant

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A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
F	T	T	F
T	T	F	T
T	T	F	T
F	F	F	T

$$A \oplus B = \boxed{(A \vee B)} \wedge \boxed{\neg (A \wedge B)}$$

XOR is Redundant

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	C		D	
$A \oplus B$	$A \lor B$	$A \land B$	$\neg(A \land B)$	$C \land D$
F	T	T	F	F
T	T	F	T	T
T	T	F	T	T
F	F	F	T	F

$$A \oplus B = \boxed{(A \lor B)} \land \boxed{\neg (A \land B)}$$

XOR is Redundant

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	C		D	
$A \oplus B$	$A \lor B$	$A \land B$	$\neg(A \land B)$	$C \land D$
F	T	T	F	F
T	T	F	T	T
T	T	F	T	T
F	F	F	T	F

$$A \oplus B = (A \lor B) \land \neg (A \land B)$$

XOR is Redundant

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DeMorgan's Rules

- Use a truth table to show that the following propositions are equivalent
- ¬ (A ∧ B) = (¬ A) ∨ (¬ B)
- I never drink wine at the beach
- It is never the case that *I drink wine* AND *I am at the beach*
- It is always the case that *I do* NOT *drink wine* OR *I am* NOT *at the beach*

Truth Table

A	B	A ∧ B	¬ (A ∧ B)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

¬ (A ∧ B) = (¬ A) ∨ (¬ B)

A	B	A ∧ B	¬ (A ∧ B)	¬ A
T	T	T	F	F
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

$$\neg (A \wedge B) = \boxed{(\neg A)} \vee (\neg B)$$

Truth Table

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A	B	A ∧ B	¬ (A ∧ B)	¬ A	¬ B
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	T	T	T

$$\neg (A \wedge B) = (\neg A) \vee \boxed{(\neg B)}$$

Truth Table

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A	B	A ∧ B	¬ (A ∧ B)	¬ A	¬ B	¬ A ∨ ¬ B
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$$\neg (A \wedge B) = \boxed{(\neg A) \vee (\neg B)}$$

Truth Table

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A	B	A ∧ B	¬ (A ∧ B)	¬A	¬ B	¬ A ∨ ¬ B
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$$\neg (A \wedge B) \boxed{=} (\neg A) \vee (\neg B)$$

Truth Table

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Truth Tables in Excel

	A	B	C	D
1	A	B	not(A and B)	(not A) or (not B)
2	FALSE	FALSE	TRUE	TRUE
3	FALSE	TRUE	TRUE	TRUE
4	TRUE	FALSE	TRUE	TRUE
5	TRUE	TRUE	FALSE	FALSE

fx =NOT(AND(A2,B2))

fx =OR(NOT(A2),NOT(B2))

DeMorgan's Rules

- Use a truth table to show that the following propositions are equivalent
  - JT: already done as an in-class exercise!
- $\neg (A \vee B) = (\neg A) \wedge (\neg B)$
- It is never the case that *I am bored* OR *tired*
- It is always the case that *I am* NOT *bored* AND NOT *tired*

- 1806-1871
- British Mathematician born in India
- Wrote more than 1000 articles!
- He introduced *Mathematical Induction* in1838

Augusts DeMorgan

- If A and B are a propositions,
- then A → B is a proposition
- that is false when A is true and B is false
- otherwise, it is true
- A → B is read *if A then B*, or *A implies B*

Implication

Implication

- If you have a Canadian passport, then you're a Canadian citizen
- A = you have a Canadian passport
- B = you're a Canadian citizen

- Logical expression:
- $A \rightarrow B$

Implication

- If you have a Canadian passport, then you're a Canadian citizen
- Maybe you have a Canadian passport (T) and you're a Canadian citizen (T)
  - JT: Can be true
- Maybe you do not have a Canadian passport (F) and you're a Canadian citizen (T)
  - JT: Can be true
- Maybe you do not have a Canadian passport (F) and you're not a Canadian citizen (F)
  - JT: Can be true
- It is not the case that (you have a Canadian passport (T) and you're not a Canadian citizen) (F)
  - JT: Claim cannot be true (i.e., it's a False claim)

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

If you have a DL, then you can drive

Truth Table Implication

- Let p be the statement: *x is in Calgary*
- Let q be the statement: *x is in Alberta*

P	Q	P->Q	
T	T	T	Calgary, Alberta : TRUE
T	F	F	Calgary, Not Alberta : FALSE
F	T	T	Not Calgary, Alberta : TRUE
F	F	T	Not Calgary, Not Alberta : TRUE

JT’s Extra Example

A	B	A → B	
T	T	T	Have pass, citizen
T	F	F	Have pass, not citizen
F	T	T	Have not pass, citizen
F	F	T	Have not pass, not citizen

Truth Table for Implication

- A → B is logically equivalent to:
  - ¬A ∨ B
  - Prove it using a truth table
- If you have a Canadian passport, then you're a Canadian citizen
- You do **not** have a Canadian passport **or** you're a Canadian Citizen

Implication is Redundant

- Using truth tables show how the result of implication can be derived with an equivalent logical expression.

JT Exercise: Show That Implication Is Redundant

- A → B is logically equivalent to:
  - ¬B → ¬A
- If you have a Canadian passport, then you're a Canadian citizen
- If you you're **not** a Canadian citizen, then you do **not** have a Canadian passport

Contrapositive

- Using truth tables show the equivalency of these logical operations.

JT Exercise: Contrapositive And Implication

- If a complex proposition does not have brackets apply operators in the following order
  1. $\neg$
  2. $\wedge$ or $\vee$ or xor, left to right
  3. $\rightarrow$

- $\neg A \wedge B = (\neg A) \wedge B$
- This is different from $\neg(A \wedge B)$
- $A \vee B \wedge C = (A \vee B) \wedge C$

Precedence

- A **tautology** is a proposition that is always true
- At the end, I will pass the course or I will not pass it
  - JT: one or the other outcome must come to pass (be true)
  - JT: Since logical-OR requires only one true input, the result is true

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

- A = I will pass the course
- $A \vee (\neg A)$

Tautologies

A	$\neg$A	$A \vee (\neg A)$
T	F	T
F	T	T

Tautologies

Contradictions

- A **contradiction** is a proposition that is always false
  - The past season, the Flames won the Stanley Cup and the Oilers won the Stanley Cup
  - If the Oilers won, then the Flames lost
  - A = The Flames won
  - $A \wedge (\neg A)$
  - JT's extra:
    - JT: one or the other outcome must come to pass (be true) while the other will not (false)
    - JT: Since logical-AND requires all inputs to be true, the result is always false

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

Contradictions

A	$\neg$ A	$A \wedge (\neg A)$
T	F	F
F	T	F

- A **contingency** is a proposition that is neither a contradiction nor a tautology

- This season, the Flames will win the Stanley Cup

Contingencies