Graphs

Peeking into Computer Science

Reading Assignment

- Mandatory: Chapter 3 – Sections 3.3 & 3.4

Graph Coloring

3

At the end of this section, you will be able to:

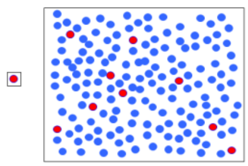
1. Understand (once more) how graphs are used to represent data
2. Learn the graph coloring algorithm
3. Apply graph coloring to solve for various scheduling problems

Objectives

Peeking into Computer Science © Jalal Kawash 2010 4

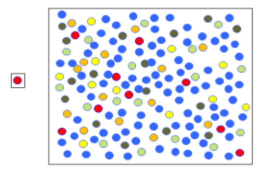
- Recall:
  - When color is used to draw attention or otherwise used to communicate information the number of colors used should be minimized.

Color Is Used Sparingly: Effective



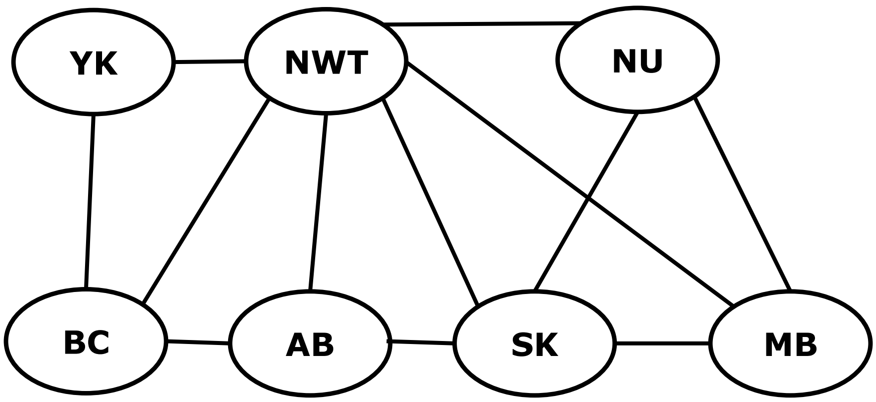
Vs.

The Increased Use Of Color: Mutes The Message

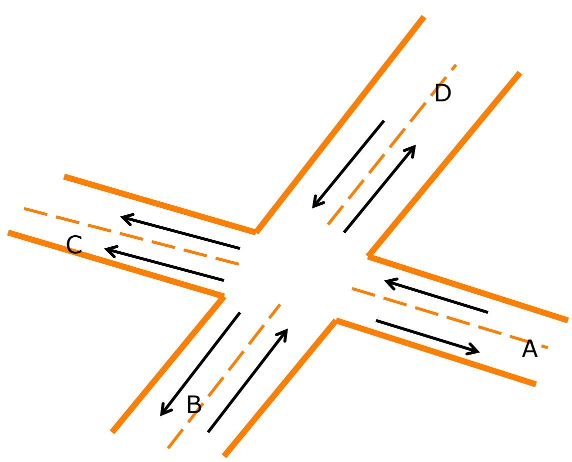


JT's Extra

JT's Extra: A Subset Of Canada: Color The Map Using As Few Colors As Possible And No Adjacent Colors



JT's Extra: Canada (As A Graph)



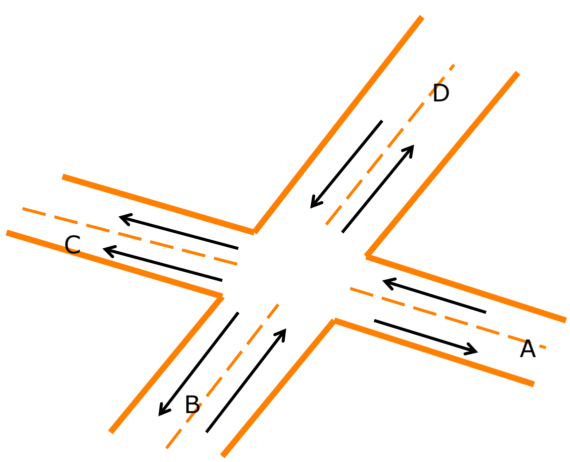
Based on Aho, Hopcroft, and Ullman, *Data Structures and Algorithms*, Addison-Wesley

Simple Intersection

- Design a traffic light for a complex intersection
- Identify allowed turns
  - Going straight is a turn!
- Group turns so that permitted simultaneous turns are in the same group
- Find the smallest number of groups
  - This means a traffic light with the smallest number of phases

Another Example Problem

Peeking into Computer Science © Jalal Kawash 2010 9

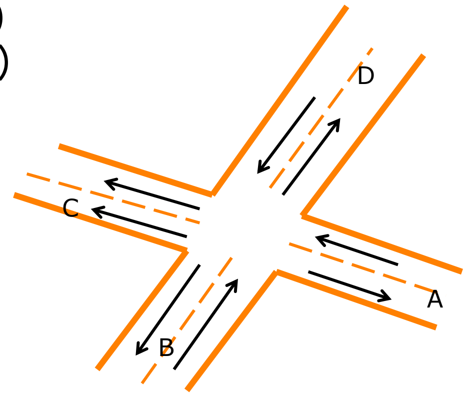


Based on Aho, Hopcroft, and Ullman, *Data Structures and Algorithms,* Addison-Wesley

Simple Intersection

Peeking into Computer Science © Jalal Kawash 2010 10

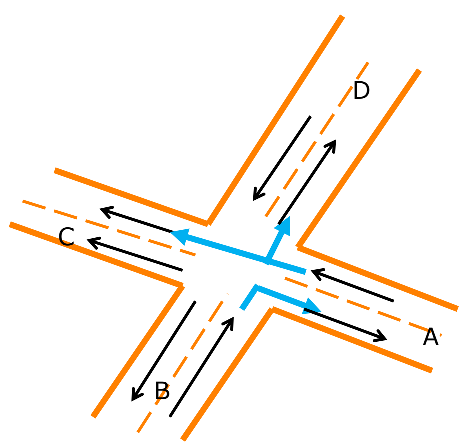
- From A to B (denoted AB)
- From A to C (AC)
- From A to D (AD)
- BA
- BC
- BD
- DA
- DB
- DC



9 Possible Turns

Peeking into Computer Science © Jalal Kawash 2010 11

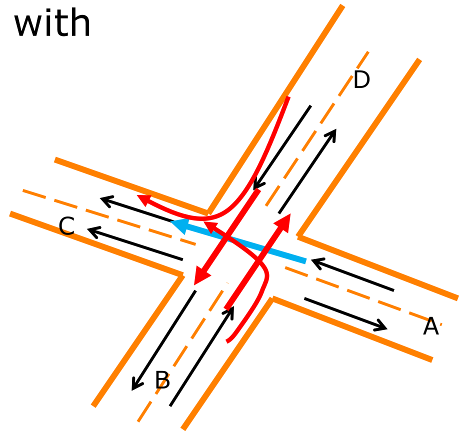
- AC
- AD
- BA



Example Simultaneous Turns

Peeking into Computer Science © Jalal Kawash 2010 12

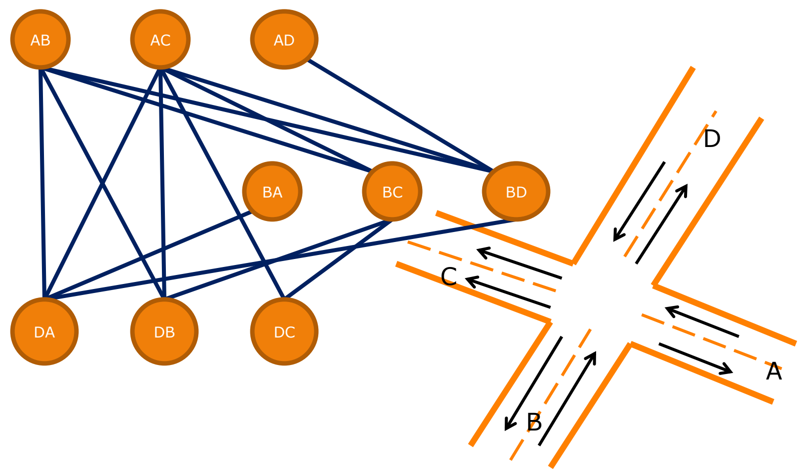
- AC conflicts with
- DB
- BD
- BC
- DC



Example Conflicting Turns

- Build a graph model
- Vertices are turns
- An edge connects conflicting turns

Objective



Graph Model of Conflicting Turns

- Color the vertices of the graph
- No two adjacent vertices can have the same color
  - Adjacent: and edge connects them
- Each color represents a group of turns that can be simultaneous
- Note: more than one solution

Graph Coloring

- Repeat the following two steps until the graph is colored
1. Select an uncolored vertex and color it with a ***new*** color, **C**
2. For each uncolored vertex,
   - Determine if it has an edge with a vertex that is colored with color **C**
   - If not, color it with color **C**
   - If yes, skip it

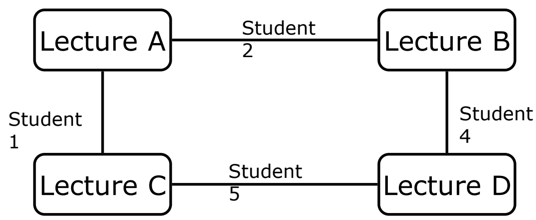
Graph Coloring Algorithm

- You are to schedule final exams so that a student will not have two final exams scheduled at the same time.
- Although you could schedule each exam in it's own individual time slot (i.e., no two exams run simultaneously) this would be highly inefficient.
  - Another constraint is to schedule exams with the minimum number of time slots.
- Examinations for a lecture will be represented with vertices.
- A pair of vertices will be connected if there is at least one student who is registered in both.

JT's Extra, Graph Coloring Example 1: Final Exams

JT's Extra, Graph Coloring Example 1: Final Exams (2)

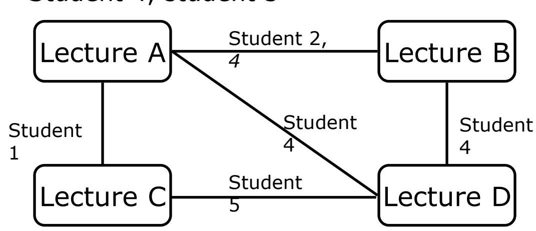
- Lecture A:
  - Student 1, student 2
- Lecture B:
  - Student 2, student 4, student 6
- Lecture C:
  - Student 1, student 3, student 5
- Lecture D:
  - Student 4, student 5



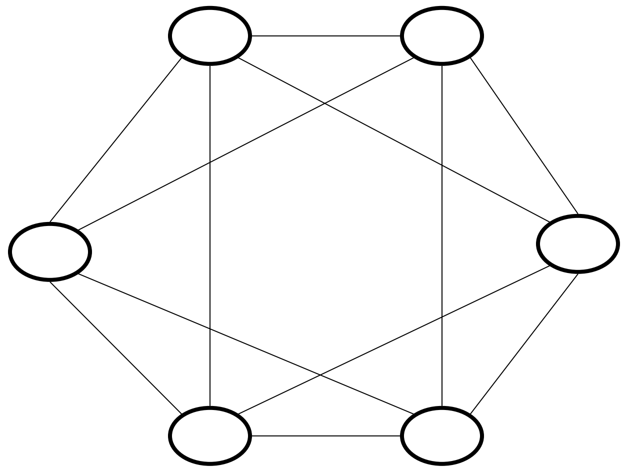
Peeking into Computer Science © Jalal Kawash 2010

JT's Extra, Graph Coloring Example 2: Final Exams

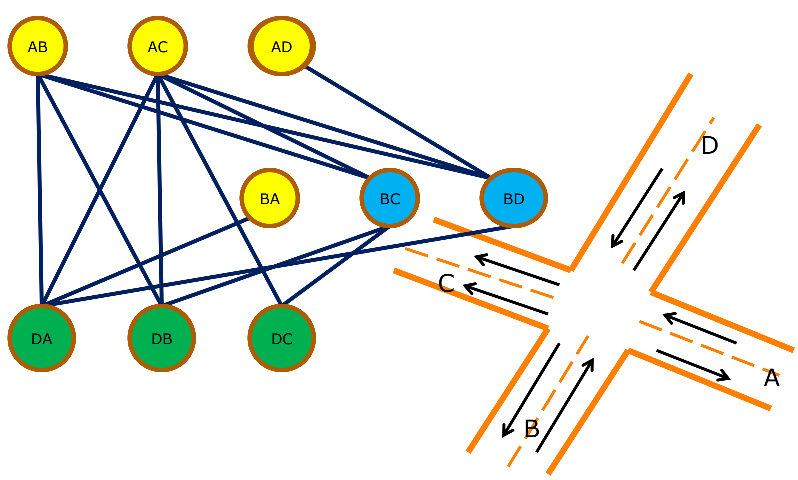
- Lecture A:
  - Student 1, student 2, *student 4*
- Lecture B:
  - Student 2, student 4, student 6
- Lecture C:
  - Student 1, student 3, student 5
- Lecture D:
  - Student 4, student 5



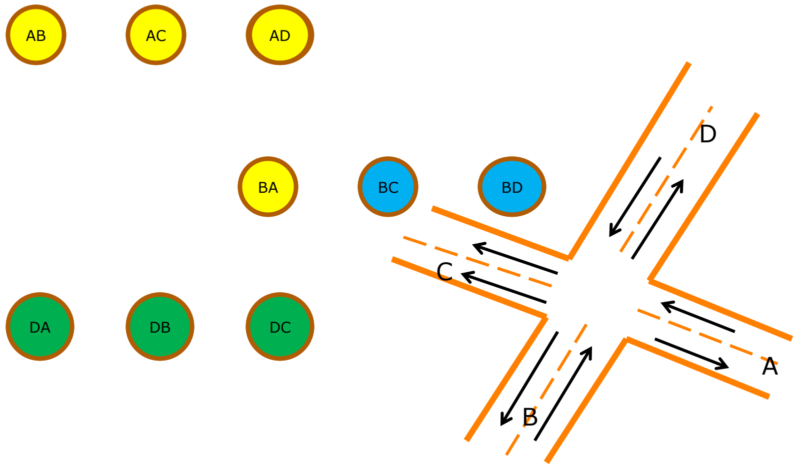
Peeking into Computer Science © Jalal Kawash 2010



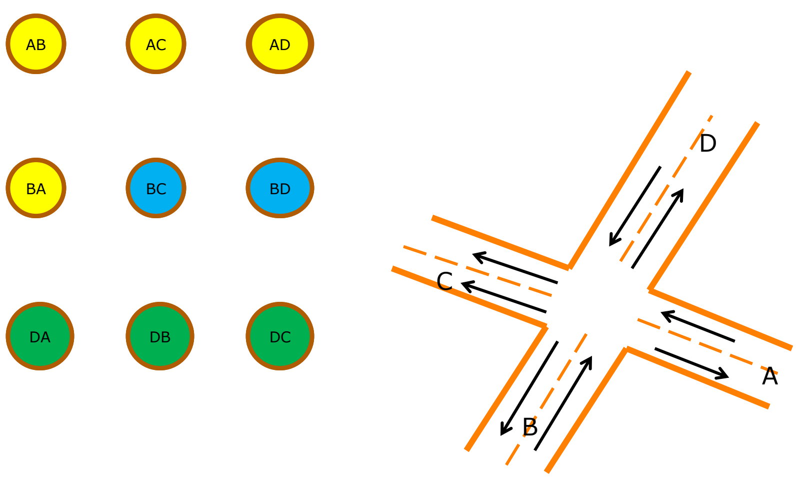
JT's Extra: Color This Graph



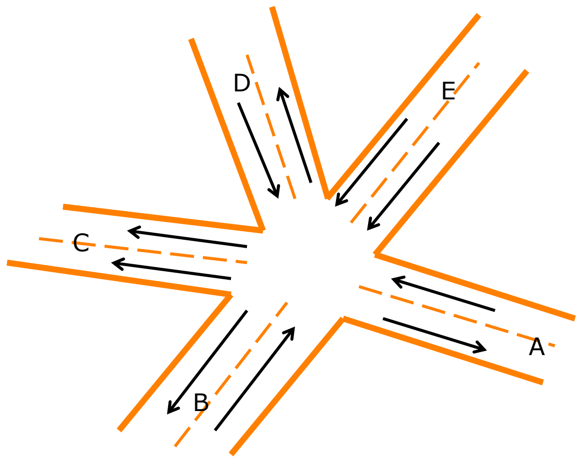
Graph Model of Conflicting Turns



Graph Model of Conflicting Turns



Graph Model of Conflicting Turns

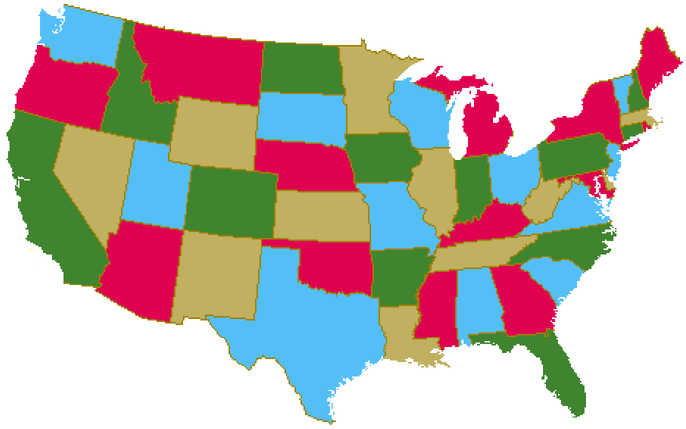


From Aho, Hopcroft, and Ullman, *Data Structures and Algorithms,* Addison-Wesley

Complex Intersection

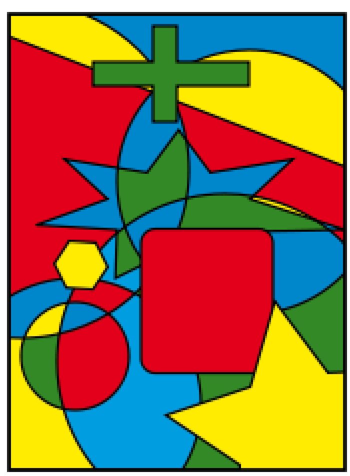
- Scheduling
- Mobile radio frequency assignment
- Sudoku

JT’s Extra: Graph Coloring Applications



JT's Extra: Complex Graphs Requiring Only 4 Colors

Peeking into Computer Science



JT's Extra: Complex Graphs Requiring Only 4 Colors

Peeking into Computer Science © Jalal Kawash 2010

Trees

A popular special case of graphs

29

At the end of this section, the student will be able to:

1. Identify when a graph is a tree
2. Use tree terminology
3. Identify different applications of trees
4. Define binary trees

Objectives

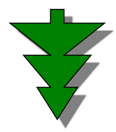
Peeking into Computer Science

Computer Science Trees

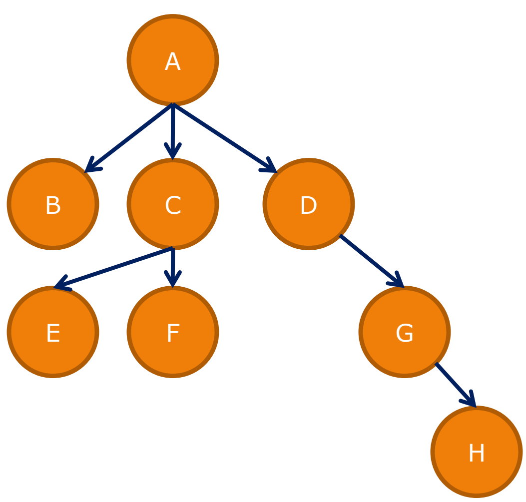
- Trees are a special case of graphs
- A *tree* is a (directed) graph with the properties:

1. There is a designated vertex, called the *root* of the tree
2. There is a unique directed path from the root to every other vertex in the tree

- Grow downwards!

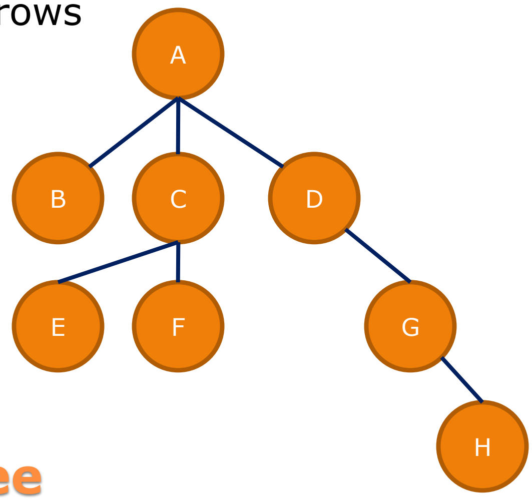


Example Tree



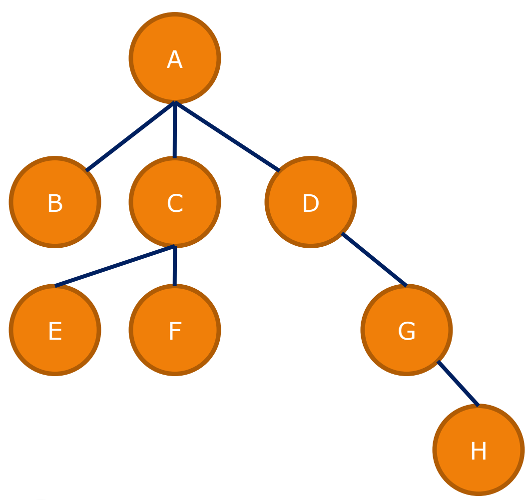
Example Tree

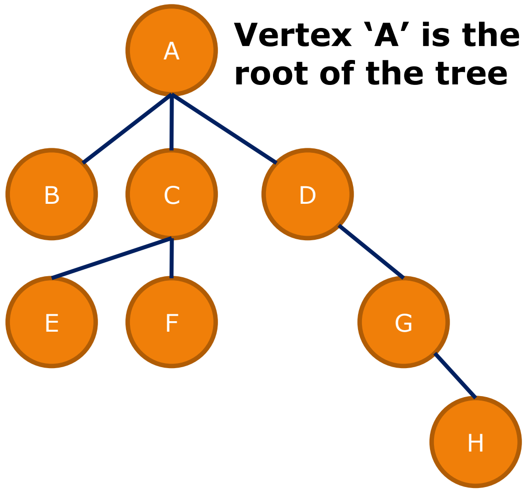
- A tree always “grows” downward
- We omit the arrows



Tree Terminology

- Root
- Child
- Parent
- Ancestor
- Descendant
- Distance
- Siblings
- Leaf

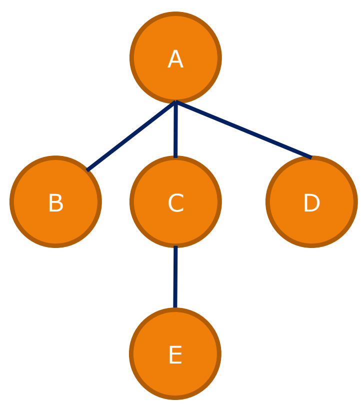




Vertex 'A' is the root of the tree

Root

- A is the parent of B, C, D.
- B, C, D are the children of A.
- B,C,D,E are descendants of A.
- A is ancestor of B, C, D, E



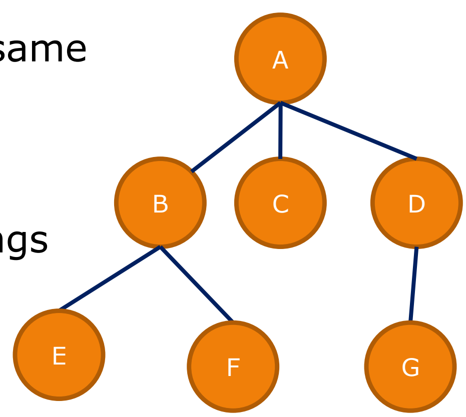
Parent-Child (Above/Below), Ancestor-Descendant

Parent-Child

- All vertices have exactly one parent except for the root (which has none).
  - JT: (by definition – the root is at the 'top' and because there are no vertices that aren't part of the tree).

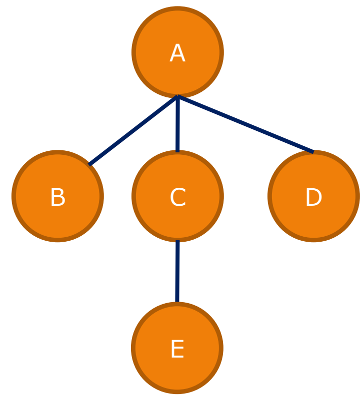
Siblings

- Siblings have the same parent.
- B,C,D are siblings
- E, F are siblings
- A, G have no siblings



Leaf

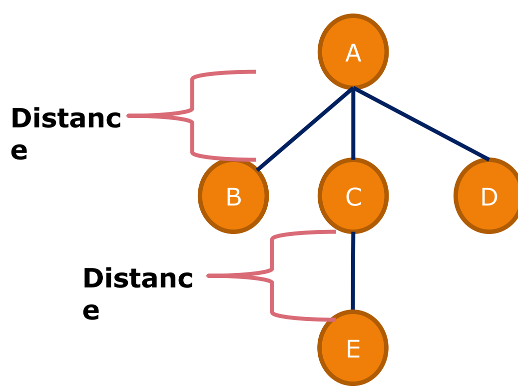
- Leaves are vertices with no children
- B, D, E are leaves
- "Bottom level"

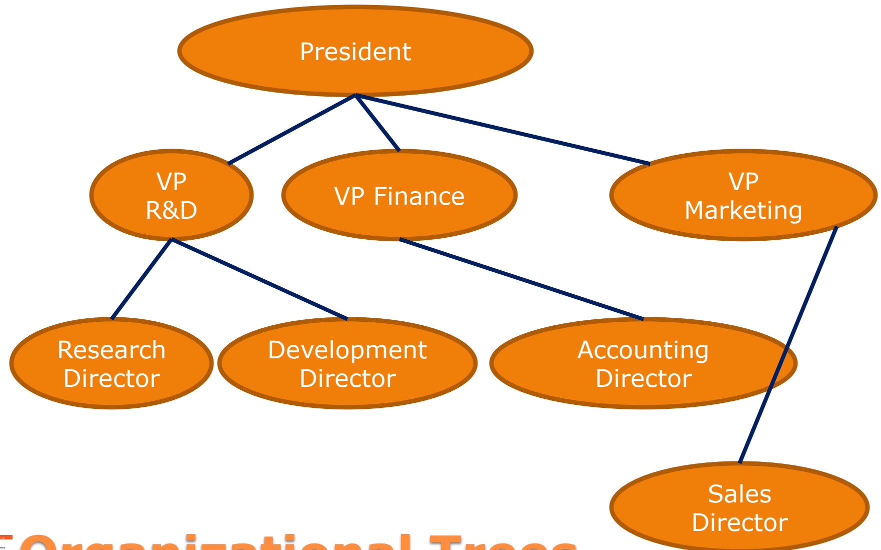


Distance

- The distance from the root to a vertex is the number of intervening edges

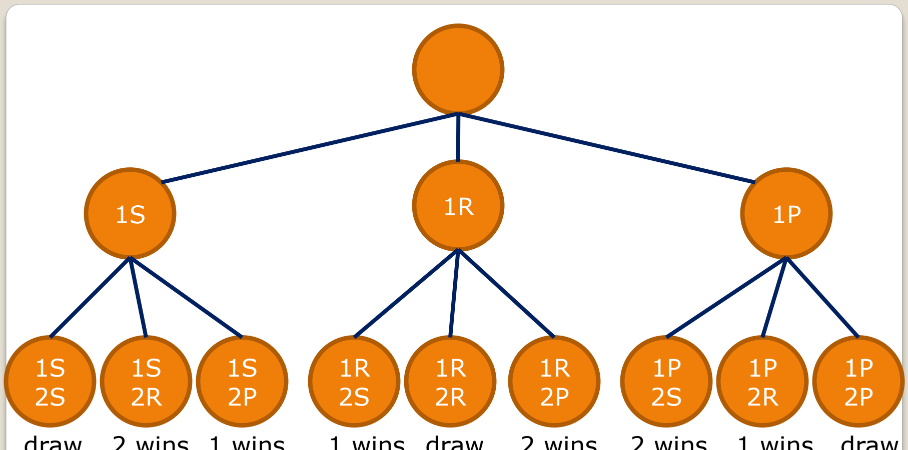
Distances
- A to B = 1
- A to E = 2





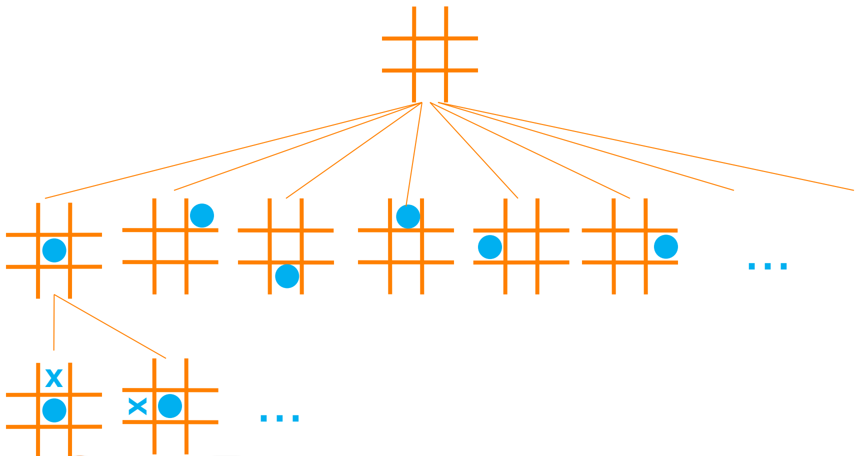
Organizational Trees

Peeking into Computer Science © Jalal Kawash 2010 41



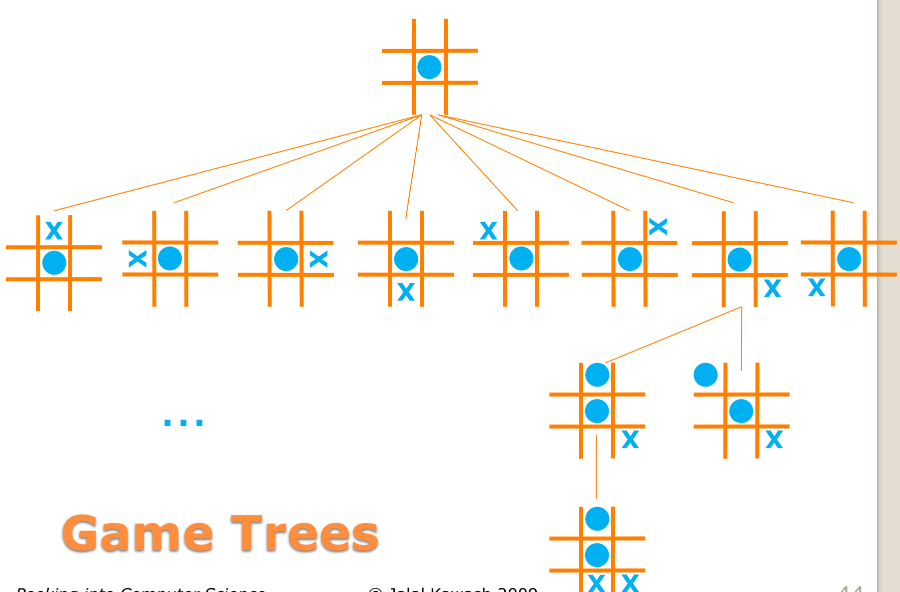
Game Trees
Paper-Rock-Scissors

Peeking into Computer Science © Jalal Kawash 2010 42



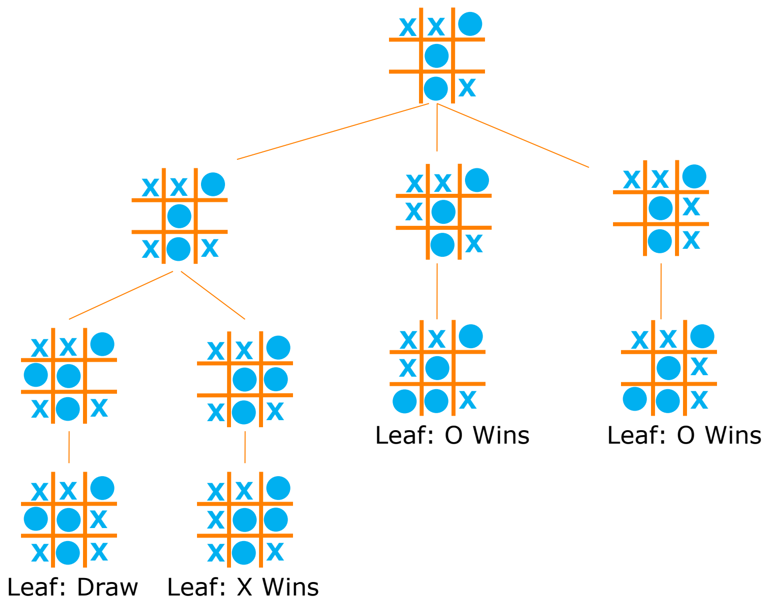
Game Trees
Tic-Tac-Toe

Peeking into Computer Science © Jalal Kawash 2009 43



Game Trees

Peeking into Computer Science © Jalal Kawash 2009 44

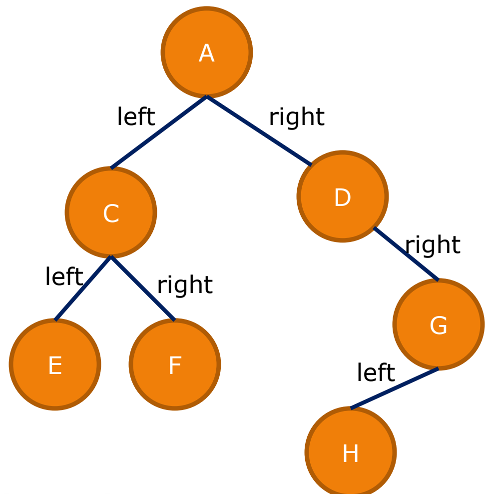


Binary Trees

- A binary tree is a tree with the following properties:

1. The edges are labeled with the labels *left* or *right*
2. Every vertex has at most two children; if both children exist, then one edge must be labeled with *left* and the other *right*.

Example Binary Tree



Example Binary Tree

- Drop the labels, if the tree is drawn with no confusion