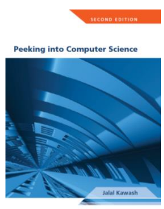
Elementary Set Theory

Peeking into Computer Science



Reading Assignment

- Mandatory: Chapter 2 – Sections 2.3 and 2.4

Relations

Relating sets

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At the end of this section, you will be able to:

1. Define relations and represent them in two ways (sets and graphical)
2. Understand the property of symmetric relations
3. Identify which relation is a function

Objectives

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JT's Extra: Reminder

- Set = $\{elem_1, elem_2 \ldots elem_n\}$
  - Order unimportant
  - E.g., colors on my shirt = {red, blue, green}
- Tuple = $(elem_1, elem_2 \ldots elem_n)$
  - Order is important
  - E.g., my ranking of favorite colors = (black, white, red, blue, green, yellow, purple)

Relations

- A relation from A to B is a subset of A x B
- Example
  - Let A = {book, lion, plate}
  - Let B = {colored, made-from-paper, has-bones, contains-glass}
  - The association of objects A with properties B is a relation from A to B
  - R = {(book, colored), (book, made-from-paper), (lion, has-bones), (plate, colored), (plate, made-from-paper), (plate, contains-glass)}
- In general, A relation on A_1 , A_2 , … , A_n is a subset of A_1 x A_2 x … x A_n
    - JT: A x B includes all possible combinations
    - JT: The relation of A to B only includes some combinations

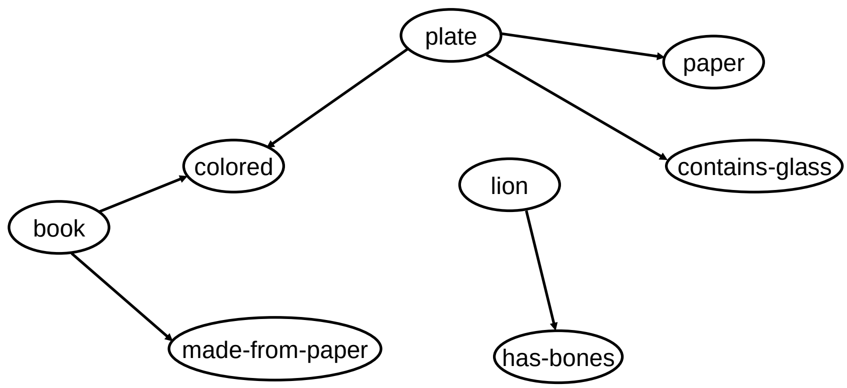
- A = {Lion, leopard}
- B = {Tan, white, spotted}

- Specify:
  - A x B, set multiplication
  - The real-life relation of the elements from A to B (i.e., only certain colour combinations possible)

JT's Extra Exercise

- Recall from the InfoViz slides (JT's extra info on when to use images, what type etc.)
- Graphical representations work well for showing how things are inter-related.
- Consequently it may be easier to show relations between sets in the form of graph (image).

JT's Extra: Relations And Pictures



JT’s Extra: Relation Graph (Previous Example)

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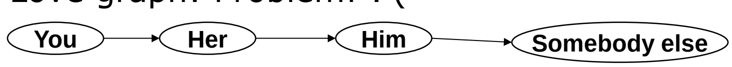
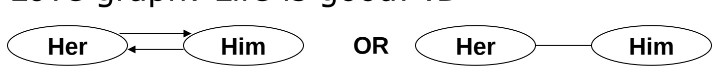
- Relations can be directed (one way) as the previous example and the example below.
  - Love graph: Problem! :’(



- Relations can also be symmetric (two way the same e.g., example below).
  - Love graph: Life is good! :D



Set Relations: Types

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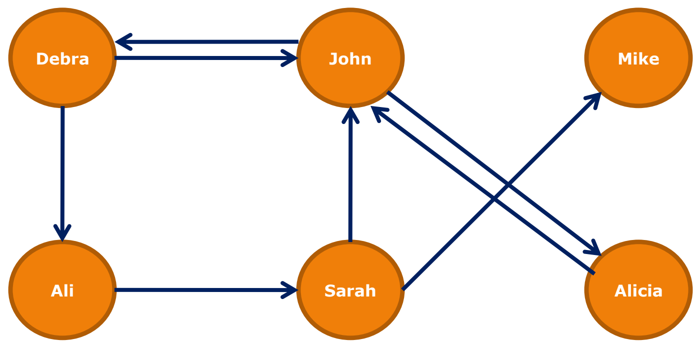
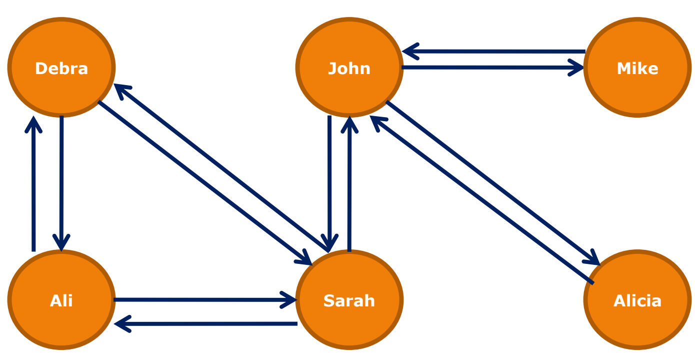


A "Likes" Relation Graph

- Whenever (a,b) ∈ R, implies that (b,a) A ∈ R, the relation is called *symmetric*



Symmetric Relations

Symmetric Relations

- Has-the-same age
- Has-the-same name
- Children of a common parent

Functions

- A function f from A to B
- Written *f:* A → B
- Is a relation from A to B, such that:
- There is exactly one pair (a,b) ∈ *f* for each a ∈ A
- We write *f*(a) = b

- Square function
  - A = {0, 1, 2, 3…}
  - B = results
  - F(a) = {0, 1, 4, 9..}

  - It is a function because for a given number in set A it will map to or produce only one result in B
  - e.g., f(2) = 4

JT’s Extra: Example Function

- With the ‘square’ function
  - f(a) = b
  - ‘a’ is the argument (input) to function ‘f’
  - ‘b’ is the return value (output/result) of function ‘f’

JT’s Extra: Terminology

- A = {1,2,3}, B={yes,no}
- {(1,yes), (2,yes), (3,no)} is a function
- {(1,yes) (2,no), (2,yes), (3,no)} is not a function
- {(2,yes), (3,no)} is not a function

Functions

- Functions “return values”

- *f*(x) = 2x
- *f*(1) returns 2
- *f*(20) returns 40

Functions

Algorithms

Computer recipes

At the end of this section, you will be able to:

1. Understand what algorithms are and how to specify them
2. Understand the importance of algorithms to computers
3. Understand what pseudo-code is
4. Develop and specify an example algorithm using pseudo-code
5. Differentiate between the correctness and efficiency of algorithms
6. Work out an example to assess the efficiency of algorithms

Objectives

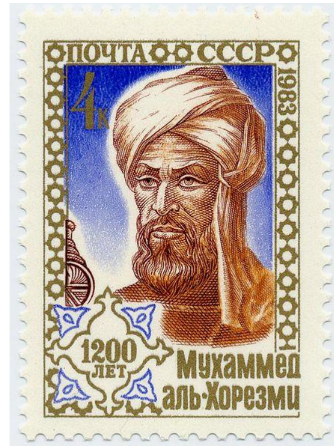
Problems

- A **problem** is a specification of the relationship between input and output
  - WHAT needs to be done, not HOW
- Cooking problem
  - Input: ingredients
  - Output: cooked food
- **Computational problems** are the ones to be carried out by computers

Algorithms

- The algorithm specifies how to solve the problem
  - How to convert the input to output
- Cooking problem
- Cooking recipe ("algorithm?")
- Algorithms are specific to Computational problems

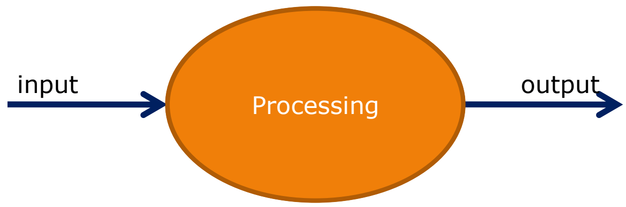
- 780-850
- Left behind the most important Math book
  - Kitab Al Jaber Wal Mokaballa
  - The book of Restoration and Comparison
  - Al Jaber (restoration) became Algebra
- When his book was translated to Greek, it was called: Thus said Algorismi
- With time, it became algorithmi



Mohamed Bin Musa Al Khawarizmi

- Was given to mathematical operations used by merchant's to simplify their accounting
- Today, it is used to mean: *computational* procedure that accepts **input**, processes it, and produces **output**

'Algorithm'



Modern 'Algorithm'

- '***algorithm***' for washing your hair

1. Wet your hair
2. Open the shampoo bottle
3. Pour 5ml of shampoo to your palm
4. Apply to hair and rub for 1 minute
5. Rinse hair with water

- Not really an algorithm, unless performed by a computer

Computational?

JT's Extra: Ways Of Specifying Algorithms

- Code (Computer program – JT's comment 'app')
  - Flash and HTML5
  - Java
  - C++
  - Etc.
- Finite State Machine (one of the later sections)
- Pseudo-code
  - English like code
- Flow charts
  - Visual representation

JT's Extra: Pseudo Code

- '***algorithm***' for washing your hair

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```
BFS(G, s)
for each vertex u ∈ V[G] − {s}
    do color[u] ← WHITE
       d[u] ← ∞
       π[u] ← NIL
color[s] ← GRAY
d[s] ← 0
π[s] ← NIL
Q ← ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    do u ← DEQUEUE(Q)
       for each v ∈ Adj[u]
           do if color[v] = WHITE
                 then color[v] ← GRAY
                      d[v] ← d[u] + 1
                      π[v] ← u
                      ENQUEUE(Q, v)
       color[u] ← BLACK
```

Image: *http://www.cs.tufts.edu/comp/150GA*

JT's Extra: Pseudo Vs. Real Code

Okay
To display the contents of a two-dimensional array onscreen you will need two nested loops and two loop controls: one to keep track of the row that is currently being displayed and one to track the current column value. Start by initializing the outer loop to the value of the lowest row value and the inner loop to the value of the lowest column value and display that array element. The inner loop will travel along the row from the lowest column value to the highest column value. After the last column has been reached, the outer loop value increases by one which allows the inner loop to traverse the second row. The outer loop will repeat until all the rows have been traversed.

PSEUDO CODE From:
www.cpsc.ucalgary.ca/~tamj

Not okay

```
for r in range (0, MAX_ROWS, 1):
   for c in range (0, MAX_COLUMNS, 1):
      sys.stdout.write(aGrid[r][c])
   print()
```

REAL CODE: From:
www.cpsc.ucalgary.ca/~tamj

Min Algorithm

- Given a set of objects (numbers)
- How do we find the smallest member of the set?
- Example input : {10, 46, 3, 100}
- Output: 3

Given input S, a set

1. Let the first element in S be *min-so-far*
2. Check if the next element is less than *min-so-far*
   - If so, make it the *min-so-far*
   - If not ignore it
3. Repeat step 2 until all elements in S have been examined
4. The minimum is *min-so-far*

Attempt

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JT's Extra: Computers Are Like Very Young Children

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Problems with the attempt

- What if the input set is empty?
- What if the input set is a *singleton* (contains one element only)?

The Min Algorithm

Input: set S
Output: minimum element in S

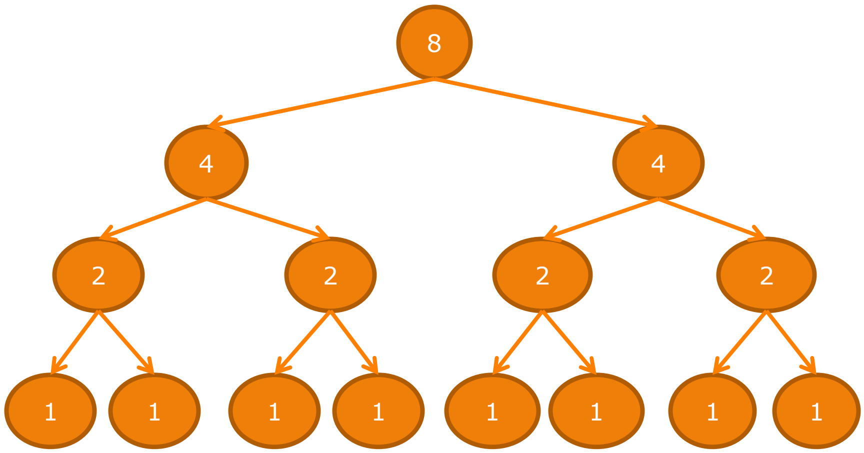
1. If S is empty, stop, there is no min
2. Otherwise, let the first element be MSF
3. Repeat the following until all elements in S have been examined:
   check if the next element is less than MSF
   - If so, make it the MSF
   - If not ignore it
4. The minimum is MSF

- Algorithms must be correct
  - Solve a problem, taking care of all special cases
- Algorithms should be efficient
  - Take less time to compute solution (time-efficiency)
  - Use less space (space-efficiency)
  - Use less power (power-efficiency)
  - Use fewer message (message-efficiency)
- Here, we only focus on time-efficiency

Correctness vs. Efficiency

- A merchant learnt that one of his 4096 golden coins is fake
- All coins look exactly the same (including the fake one)
- The only difference between a fake and a real coin is in the weight
- A fake coin is lighter
- Find an efficient algorithm to locate the fake coin
- Hint: Can be done in 12 comparisons

Example



A partial tree

- If you compare each pair of coins, you may need 4096/2 = 2048 "operations"

1. Weigh 2048 against 2048, pick the lighter pile
2. Weigh 1024 against 1024, pick the lighter pile
3. Weigh 512 against 512, pick the lighter pile
4. Weigh 256 against 256, pick the lighter pile
5. Weigh 128 against 128, pick the lighter pile
6. Weigh 64 against 64, pick the lighter pile
7. Weigh 32 against 32, pick the lighter pile
8. Weigh 16 against 16, pick the lighter pile
9. Weigh 8 against 8, pick the lighter pile
10. Weigh 4 against 4, pick the lighter pile
11. Weigh 2 against 2, pick the lighter pile
12. Weigh 1 against 1, pick the lighter **coin**

Efficient Algorithms

- It can be shown that the height of this binary tree is $\mathbf{log_2 n}$, where n is the number of coins
- $\mathbf{log_2 4096 = 12}$
- **@#$$&^&)(&*^$##$%^$#**

Logarithmic Time

