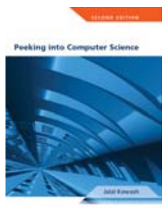
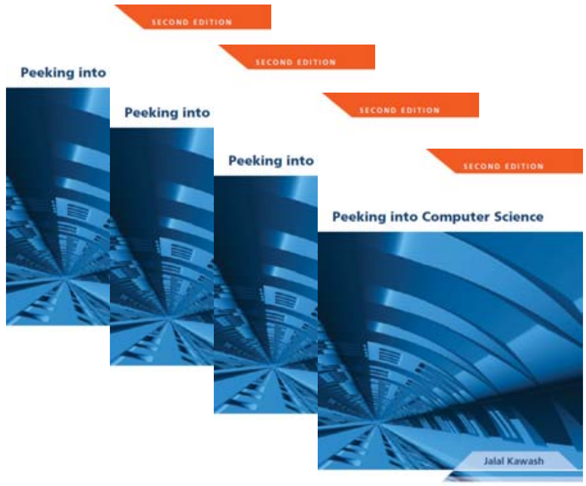
Logic

Peeking into Computer Science



Reading Assignment

- Mandatory: Chapter 2 – Section 2.1



Propositional Logic

By the end of this section, you will be able to:

1. Define a proposition
2. Define and combine the logic operators: AND, OR, NOT, XOR, and implies
3. Use truth tables to determine equivalence of propositions
4. Determine if a proposition is a tautology, contradiction, or contingency

Objectives

Propositions and Truth Values

- A proposition is a statement whose value is either TRUE or FALSE

- It is snowing today
- I am not older than you
- Canada is the largest country
- Canada shares a border with the US

Not Propositions

- X > 12
- Mr. X is taller than 250cm
- X+Y = 5
- Mrs. Y weighs 50kg

- Unless we know what the values of X and Y are, these are not propositions

- The number 5
- lol!

JT's Extra: Not Propositions

- Propositions can be built from other propositions using logical operators:
  - AND, OR, NOT, and XOR (exclusive OR)
- It is raining today **AND** it is very warm
- At 12:00 today, I will be eating **OR** I will be home (inclusive OR)
- I will be either at the beach **OR** hiking (XOR)
- I will **NOT** be home at 6

Compound propositions

- The popular usage of the logical AND applies when *ALL* conditions must be met (very stringent).
  - In terms of logic: 'AND' applies when all propositions must be true.
  - Example: I picked up all the kids today.
  - Picked up son AND picked up daughter.

JT's Extra: Logical AND

- The correct everyday usage of the logical OR applies when *ATLEAST* one condition must be met (less stringent).
  - In terms of logic: 'OR' applies when one or more propositions are true.
  - Example:
  - Using the 'Peeking' book OR using the 'Alice' book

JT's Extra: Logical OR

JT's Extra: Logical NOT

- The everyday usage of logical NOT negates (or reverses) a statement.
- In terms of logic: 'NOT' reverses a proposition (true becomes false and false becomes true).
- I am finding this class quite stimulating and exciting....*NOT!!!*

Statement (logical condition) — "I am finding this class quite stimulating and exciting"

Negation of the statement/condition — "NOT!!!"

Example: U of C Calendar

- Bachelor of Commerce (Year 1) Required Grade 12 High School Subject
  - English 30 or ELA 30-1 and
  - Pure Mathematics 30 and
  - Subject from Group A or B

Eligible tuition fees

Generally, a course qualifies if it was taken at the post-secondary level or (for individuals aged 16 or over at the end of the year) it develops or improves skills in an occupation and the educational institution has been certified by Human Resources and Social Development Canada. In addition, you must have taken the course in 2007.

Example: Income Tax Guide

Line 349 - Donations and gifts

You can claim donations either you or your spouse or common-law partner made. For more information about donations and gifts, or if you donated any of the following:

- gifts of property other than cash; or
- gifts to organizations outside Canada; or
- gifts to Canada, a province, or a territory made after 1997 and agreed to in writing before February 19, 1997.

Example: Income Tax Guide

- If A is a proposition,
- then ¬A is a proposition
- that is true when A is false
- And false when A is true

- ¬A is read *NOT A*

Negation (JT 'NOT')

A	¬ A
T	F
F	T

It is raining today

It is **not** raining today

Truth Table for Negation

- If A and B are a propositions,
- then A $\wedge$ B is a proposition
- that is true when both A and B are true
- otherwise, it is false

- A $\wedge$ B is read *A and B*

Conjunction (JT: 'AND')

- If A and B are a propositions,
- then A $\vee$ B is a proposition
- that is false when both A and B are false
- otherwise, it is true

- A $\vee$ B is read *A or B*

Disjunction (JT: 'OR')

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

It is raining today **and** it is cold

JT: Column A **JT: Column B**

Truth Table for Conjunction

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

It is raining today **or** it is cold

JT: Column A **JT: Column B**

Truth Table for Disjunction

Inclusive Vs. Exclusive OR

- JT's Extra: sometimes the everyday usage of 'OR' does not correspond to 'Logical OR'.
- To be eligible for the job, you have to be above 21 ***or*** attending a post-secondary institution
- We'll drive you to your place; my friend ***or*** I will be driving your car

Exclusive OR

- If A and B are a propositions,
- then $A \oplus B$ is a proposition
- that is true when exactly one of A and B is true
- otherwise, it is false
- $A \oplus B$ is read *A xor B*

A	B	A ⊕ B
T	T	F
T	F	T
F	T	T
F	F	F

It will be at home **or** at school

Truth Table for XOR

A	B	A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
T	T	F	T	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	F	F	T

A xor B = (A or B) and not (A and B)

XOR is Redundant

A	B	A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
T	T	F	T	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	F	F	T

$$A \oplus B = (A \vee B) \wedge \neg (A \wedge B)$$

XOR is Redundant

A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)
F	T	T	F
T	T	F	T
T	T	F	T
F	F	F	T

$$A \oplus B = (A \vee B) \wedge \neg (A \wedge B)$$

XOR is Redundant

	C		D	
A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)	C ∧ D
F	T	T	F	F
T	T	F	T	T
T	T	F	T	T
F	F	F	T	F

$$A \oplus B = (A \vee B) \wedge \neg (A \wedge B)$$

XOR is Redundant

	C		D	
A ⊕ B	A ∨ B	A ∧ B	¬(A ∧ B)	C ∧ D
F	T	T	F	F
T	T	F	T	T
T	T	F	T	T
F	F	F	T	F

$$A \oplus B = (A \vee B) \wedge \neg (A \wedge B)$$

XOR is Redundant

- Use a truth table to show that the following propositions are equivalent
- $\neg (A \wedge B) = (\neg A) \vee (\neg B)$
- I never drink wine at the beach
- It is never the case that *I drink wine* AND *I am at the beach*
- It is always the case that *I do* NOT *drink wine* OR *I am* NOT *at the beach*

DeMorgan's Rules

A	B	$A \wedge B$	$\neg (A \wedge B)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\neg (A \wedge B) = (\neg A) \vee (\neg B)$

Truth Table

A	B	A ∧ B	¬ (A ∧ B)	¬ A
T	T	T	F	F
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

$$\neg (A \wedge B) = (\neg A) \vee (\neg B)$$

Truth Table

A	B	A ∧ B	¬ (A ∧ B)	¬ A	¬ B
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	T	T	T

$$\neg (A \wedge B) = (\neg A) \vee (\neg B)$$

Truth Table

Truth Table

A	B	A ∧ B	¬ (A ∧ B)	¬ A	¬ B	¬ A ∨ ¬ B
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$\neg (A \wedge B) = (\neg A) \vee (\neg B)$

Truth Table

A	B	A ∧ B	¬ (A ∧ B)	¬A	¬ B	¬ A ∨ ¬ B
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$\neg (A \wedge B) = (\neg A) \vee (\neg B)$

	A	B	C	D
1	**A**	**B**	**not(A and B)**	**(not A) or (not B)**
2	FALSE	FALSE	TRUE	TRUE
3	FALSE	TRUE	TRUE	TRUE
4	TRUE	FALSE	TRUE	TRUE
5	TRUE	TRUE	FALSE	FALSE

=NOT(AND(A2,B2))

=OR(NOT(A2),NOT(B2))

Truth Tables in Excel

- Use a truth table to show that the following propositions are equivalent
- $\neg (A \vee B) = (\neg A) \wedge (\neg B)$
- It is never the case that *I am bored* OR *tired*
- It is always the case that *I am* NOT *bored* AND NOT *tired*

DeMorgan's Rules

- 1806-1871
- British Mathematician born in India
- Wrote more than 1000 articles!
- He introduced *Mathematical Induction* in1838

Augusts DeMorgan

- If A and B are a propositions,
- then A → B is a proposition
- that is false when A is true and B is false
- otherwise, it is true

- A → B is read *if A then B*, or *A implies B*

Implication

Implication

- If you have a Canadian passport, then you're a Canadian citizen
- A = you have a Canadian passport
- B = you're a Canadian citizen

- Logical expression:
- $A \rightarrow B$

Implication

- If you have a Canadian passport, then you're a Canadian citizen
- Maybe you have a Canadian passport (T) and you're a Canadian citizen (T)
  - JT: Can be true
- Maybe you do not have a Canadian passport (F) and you're a Canadian citizen (T)
  - JT: Can be true
- Maybe you do not have a Canadian passport (F) and you're not a Canadian citizen (F)
  - JT: Can be true
- It is not the case that (you have a Canadian passport (T) and you're not a Canadian citizen) (F)
  - JT: Claim cannot be true (i.e., it's a False claim)

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

If you have a DL, then you can drive

Truth Table Implication

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

Have pass, citizen

Have pass, not citizen

Have not pass, citizen

Have not pass, not citizen

Truth Table for Implication

Implication is Redundant

- $A \rightarrow B$ is logically equivalent to:
  - $\neg A \vee B$
  - Prove it using a truth table
- If you have a Canadian passport, then you’re a Canadian citizen
- You do **not** have a Canadian passport **or** you’re a Canadian Citizen

Contrapositive

- $A \rightarrow B$ is logically equivalent to:
  - $\neg B \rightarrow \neg A$
- If you have a Canadian passport, then you’re a Canadian citizen
- If you you’re **not** a Canadian citizen, then you do **not** have a Canadian passport

- If a complex proposition does not have brackets apply operators in the following order
  1. $\neg$
  2. $\wedge$ or $\vee$ or xor, left to right
  3. $\rightarrow$

- $\neg A \wedge B = (\neg A) \wedge B$
- This is different from $\neg(A \wedge B)$
- $A \vee B \wedge C = (A \vee B) \wedge C$

Precedence

- A **tautology** is a proposition that is always true

- At the end, I will pass the course or I will not pass it
- A = I will pass the course
- $A \vee (\neg A)$

Tautologies

A	$\neg$A	A $\vee$($\neg$ A)
T	F	T
F	T	T

Tautologies

- A **contradiction** is a proposition that is always false
- The past season, the Flames won the Stanley Cup and the Oilers won the Stanley Cup
- If the Oilers won, then the Flames lost
- A = The Flames won
- A $\wedge$ ($\neg$ A)

Contradictions

Contradictions

A	¬ A	A ∧(¬A)
T	F	F
F	T	F

Contingencies

- A **contingency** is a proposition that is neither a contradiction nor a tautology
- This season, the Flames will win the Stanley Cup