Basic Set Theory

You will learn basic properties of sets and set operations.

What Is A Set?

- A collection of elements/members.
- Example: students in this lecture.

Representing Sets

- Small sets can be represented by showing all the members.
  - Example:

  family = {mother, father, older brother, younger brother, little sister}
- Larger sets may be difficult to represent (infinite) so a notation must be used to specify the conditions for membership.
  - Examples:

  A = {x | x is a citizen in Canada}

  B = {x | x is an even number}
- Representing set membership: $\in$
  - Example:

  James Tam $\in$ {Canadian citizen}

Sets That Contain No Elements

- An empty set contains no elements.
- Notation:

  A = {}

  A = ϕ

Important Characteristics Of Sets

- Order
- Duplication

Order

- Generally order isn't important for sets
  - Example:
    {Mother, Father, Daughter}
    Is the same as
    {Father, Mother, Daughter}
- A tuple is special type of set where order is important and is denoted with round brackets instead of curly braces.
  - (Alice, Bob, Charley) is not the same as (Bob, Charley, Alice)

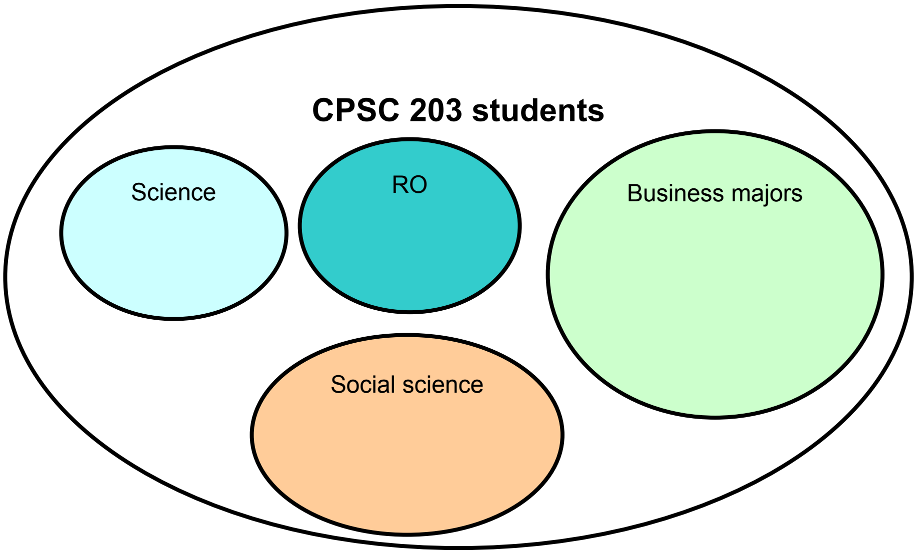
Duplicates

- Duplicate elements may or may not be allowed.
- Generally for most sets duplicates are not allowed.
  {Father, Father, Mother, Daughter}
  - Should be
  {Father, Mother, Daughter}
- Multi-sets: the case that does allow for duplicates
  - {Larry, Darryl, Darryl}

Subset

- All the elements of one set (subset) that are also elements of another set (superset)
- Example:
  - Women who live in Canada (subset), People who live in Canada (superset).
- Notation:
  - *Subset* $\subseteq$ *Super set*
  {1} $\subseteq$ {1,2,3}
- A set is also a subset of itself
  {1,2,3} $\subseteq$ {1,2,3}
- The empty set is also a subset of any set

Venn Diagrams: Subsets



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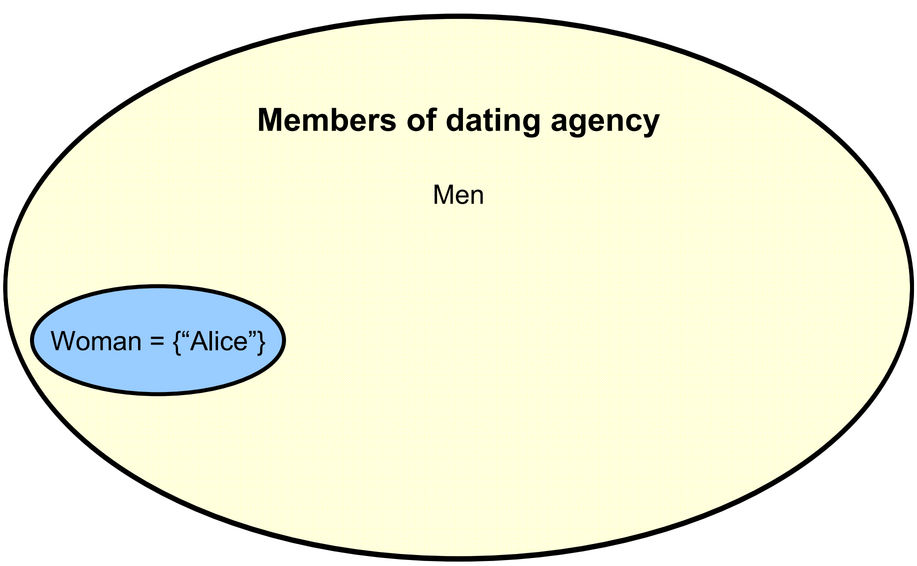
Venn Diagrams: Subsets



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Set Operations

1. Intersection
2. Union
3. Subtraction
4. Multiplication (Book: Cartesian product)

Set Intersection

- Elements that are members of two sets.
- Elements of one set AND elements of another set.
  - Example:

    A = {1, 2, 3, 4}
    B = {3, 4, 5}
    $A \cap B$ = C, C = {3, 4}

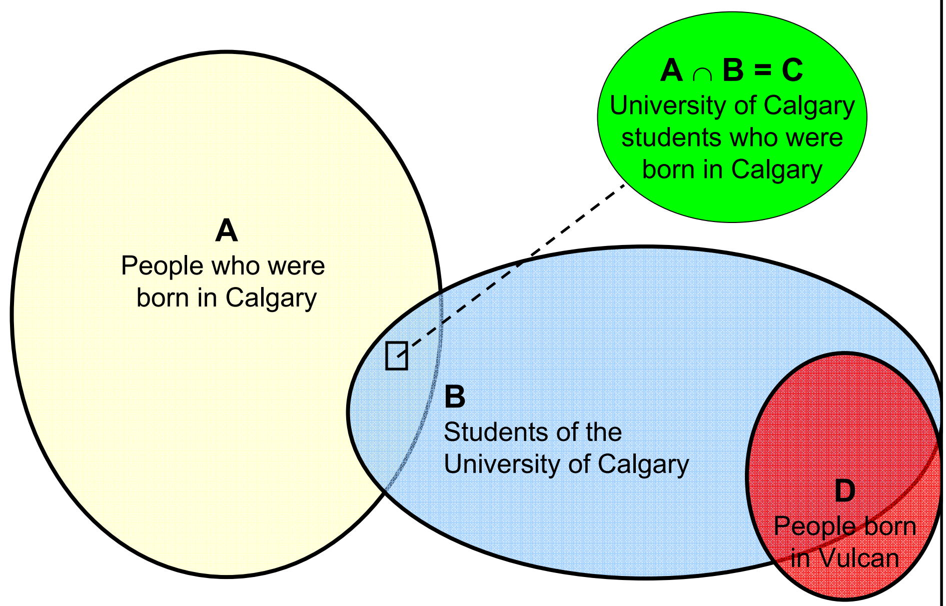
  - Example:

    A = {0, 2, 4, 6, 8}
    B = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
    $A \cap B$ = {0, 2, 4, 6, 8} = B

  - Example:

    A = {2, 4, 6, 8..} (positive even integers)
    B = {1, 3, 5, 7…} (positive odd integers)
    $A \cap B$ = {} (Disjoined sets)

Venn Diagrams: Set Intersection



Set Union

- The elements of two sets combined.
- Includes elements that are in one set OR the other set.
  - Example

    A = {1, 2, 4}

    B = {1, 2, 3}

    $A \cup B$ = {1, 2, 3, 4}

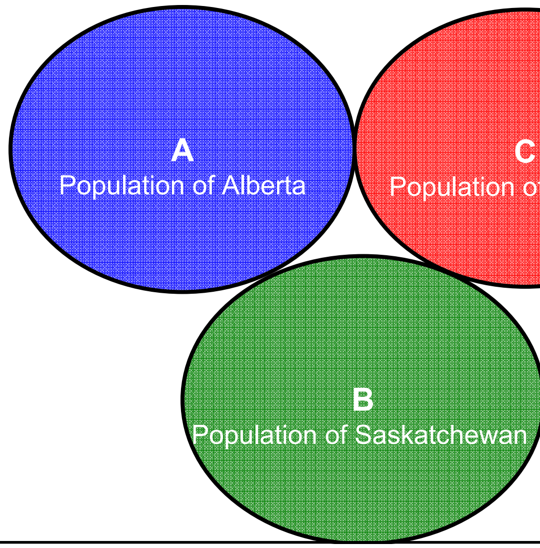
  - Example

    A = {2, 4, 6, 8..} (positive even integers)

    B = {1, 3, 5, 7…} {positive odd integers)

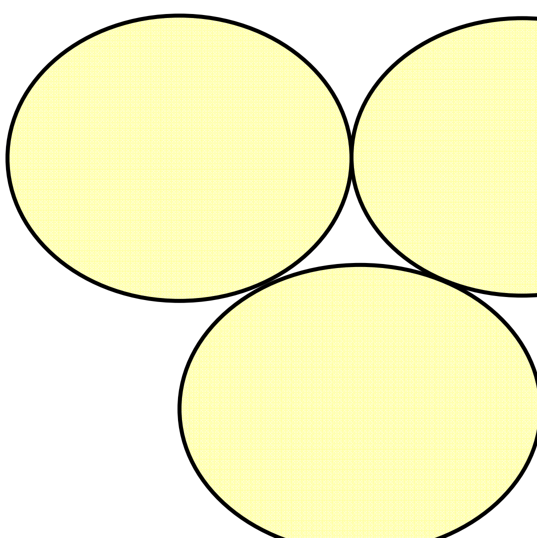
    $A \cup B$ = {1, 2, 3, 4, 5…} (positive integers)

Venn Diagram: Set Union



Venn Diagram: Set Union

$A \cup B \cup C = D$ (Population of the prairie provinces)



Set Subtraction

- Take out the elements of one set that are in another set
  - Example

    A= {12, 1, 2, 23}

    B = {0, 1, 2, 3, 4, 5}

    A – B = {12, 23}

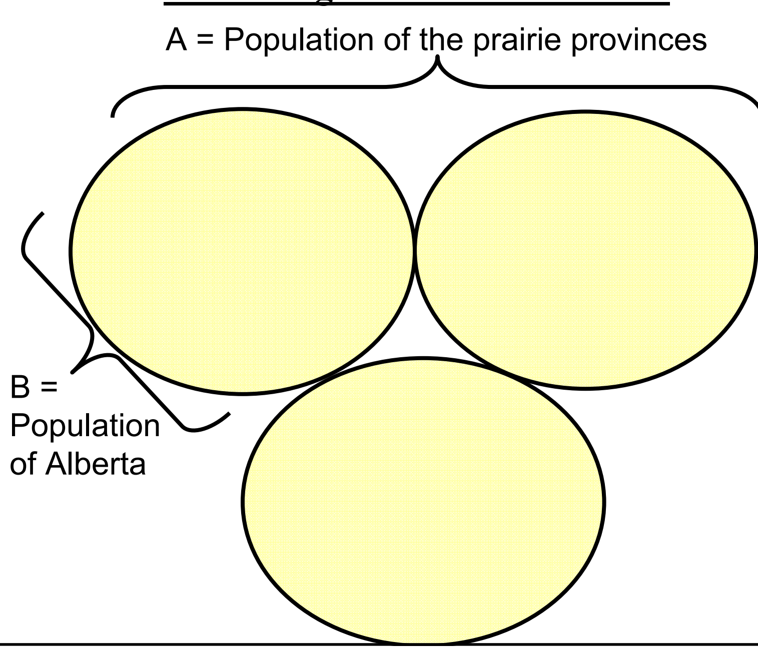
- Set subtraction of a superset from a subset yields the empty set.
  - Example

    A = {1, 3, 5}

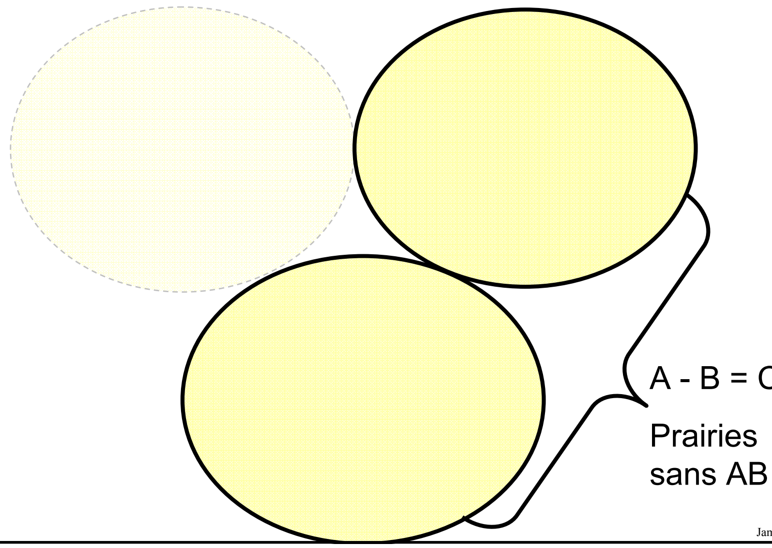
    B = {all positive integers}

    A – B = {}

Venn Diagram: Set Subtraction



Venn Diagram: Set Subtraction



Set Multiplication

- "Takes all combinations from the sets"
- (If you prefer a Mathematical definition – from the lecture notes of Jalal Kawash): $A_1 \times A_2 \times \ldots \times A_n = \{(a_1,a_2,\ldots, a_n) \mid a_1 \text{ is in } A_1 \text{ and } a_2 \text{ is in } A_2 \ldots a_n \text{ is in } A_n\}$
- The operation may be used in decision making to ensure that all combinations have been covered.

Set Multiplication: Applications

- Developing a game where all combinations must be considered in order to determine the outcome.
- Each combination is a tuple (not a set).

A = {player one, player two}
B = {rock, paper, scissors}
A x B = {(player one, rock), (player one, paper), (player one, scissors),
(player two, rock), (player two, paper), (player two, scissors)}

- (Examples from actual software will be much more complex and taking a systematic approach helps ensure that nothing is missed).

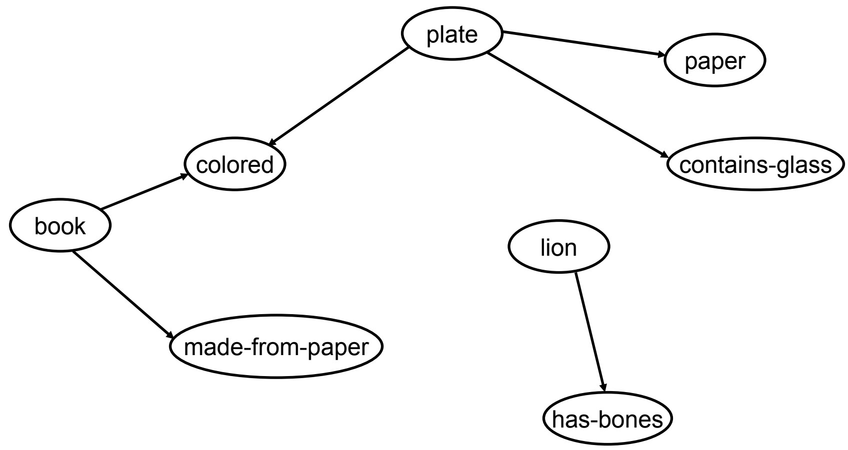
A = {player one, player two, player three...}
B = {completed quest one, completed quest two...}
C = {healthy, injured, poisoned, diseased, dead, gone forever}

Set Relations

- Can be used to show how elements of a set or sets connect (or don’t connect).
- Relationships between the elements of different sets produces another set (of tuples) that show the relations.
  - Example (from page 31 of the text).

O set of objects = {book, lion, plate}
P set of properties = {colored, made-from-paper, has-bones, contains-glass}
R set of relations from set O to P = {(book, colored), (book, made-from-paper), (lion, has-bones), (plate, colored), (plate, paper), (plate, contains-glass)}

Venn Diagram: Set Relations



Set Relations: Types

- Relations can be directed (one way) as the previous example.
- Relations can also be symmetric (two way – graphs, next section).

You Should Now Know

- What is a set
- How to textually specify a set and how to represent sets using a Venn diagram
- What is an empty set
- What is the difference between a set, a tuple and a multi-set
- What is a subset and what is a superset
- Common set operations: intersection, union, subtraction, multiplication (Cartesian product)
- What is a set relation