Numerical Representations On The Computer: Negative And Rational Numbers

- **How are negative and rational numbers represented on the computer?**
- **How are subtractions performed by the computer?**

Subtraction

- In the real world
  A - B
- In the computer
  A - B

Subtraction

- In the real world
  A - B
- In the computer
  A - B
  A + (-B)

Not done this way!

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Binary Subtraction

- Requires the complementing of a binary number
  - i.e., A – B becomes A + (-B)
- The complementing can be performed by representing the negative number as a ones or twos complement value.

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Complementing Binary Using The Ones Complement Representation

- For positive values there is no difference (no change is needed)
  - e.g., positive seven (The 'A' in the expression A – B)

    0111 (regular binary)

    0111 (Ones complement equivalent)
- For negative values complement the number by negating the binary values: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0).
  - e.g., minus six (The 'B' in the expression A – B becomes A+(-B))

    -0110 (regular binary)

    1001 (Ones complement equivalent)

Complementing Binary Using The Twos Complement Representation

- For positive values there is no difference (no change is needed)
  - e.g., positive seven (The 'A' in the expression A – B)

    0111 (regular binary)

    0111 (Twos complement equivalent)
- For negative values complement the number by negating the number: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0) *and adding one to the result*.
  - e.g., minus six (The 'B' in the expression A – B becomes A+(-B))

    -0110 (regular binary)

    1010 (Twos complement equivalent)

Interpreting The Bit Pattern: Complements

- Recall:
  - Positive values remain unchanged:
    - 0110 is the same value with all three representations.
  - Negative values are converted through complementing:
    - Ones complement: negate the bits
      -0110 becomes 1001
    - Twos complement: negate the bits and add one
      -0110 becomes 1010
- Problem: the sign must be retained (complements don't use a minus sign).
- Approach:
  - One bit (most significant bit/MSB or the signed bit) is used to indicate the sign of the number.
  - This bit cannot be used to represent the magnitude of the number
  - If the MSB equals 0, then the number is positive
    e.g. 0 bbb is a positive number (bbb stands for a binary number)
  - If the MSB equals 1, then the number is negative
    e.g. 1 bbb is a negative number (bbb stands for a binary number)

Summary Of The Three Binary Representations

	Positive values are represented with:	Negative values are represented with:
Regular binary	No explicit symbol is needed (rarely is a plus '+' used) e.g., 100 vs. +100	A minus '-' sign e.g., **-**100
Ones complement	The sign bit (MSB) is zero e.g., **0**11	The sign bit (MSB) is one e.g., **1**00
Twos complement	The sign bit (MSB) is zero e.g., **0**11	The sign bit (MSB) is one e.g., **1**00

Interpreting The Bits: All Representations

Bit pattern	Regular binary	Ones complement	Twos complement
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

What You Already Should Know

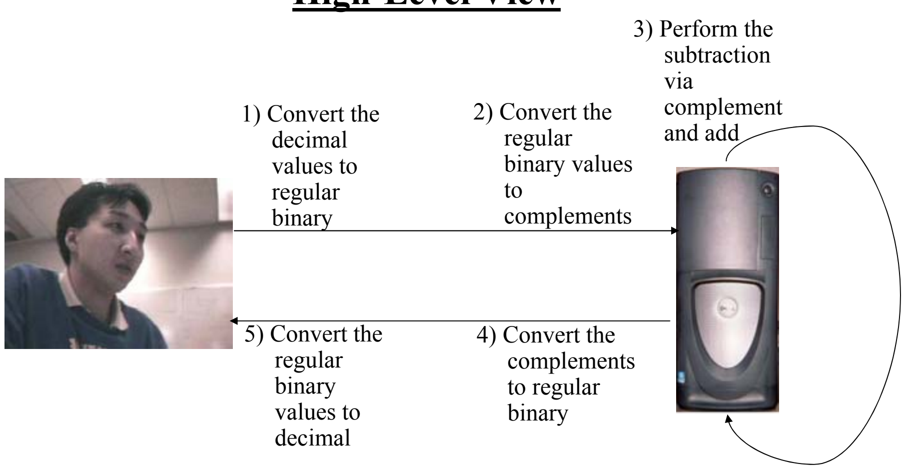
- How to convert from decimal to binary.
- How to convert from binary to decimal.

What You Will Learn

- How to subtract numbers with the complement and add technique:
  The operation A – B is performed as A + (-B)

James Tam

Binary Subtraction Via Complement And Add: A High-Level View



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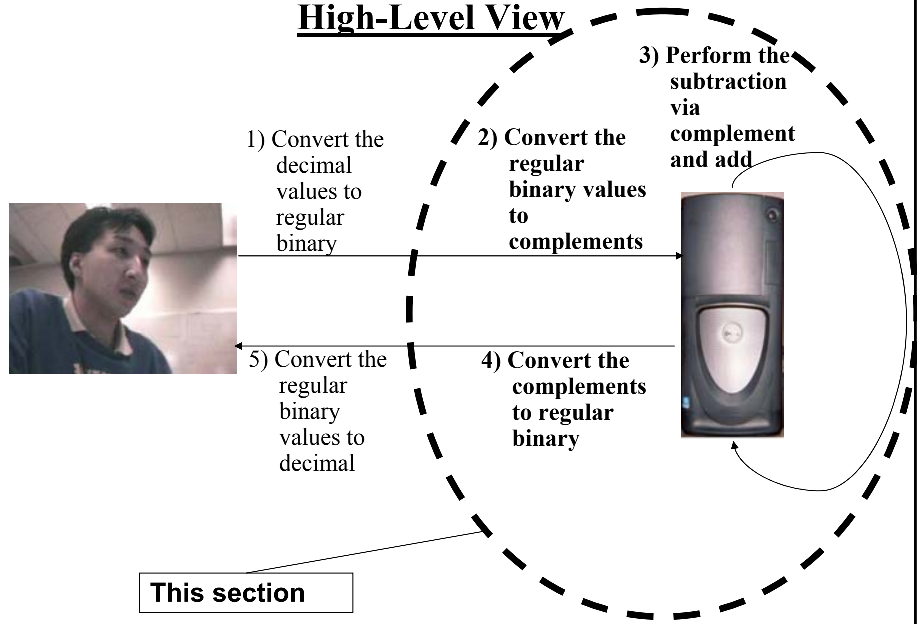
Binary Subtraction Via Complement And Add: A High-Level View



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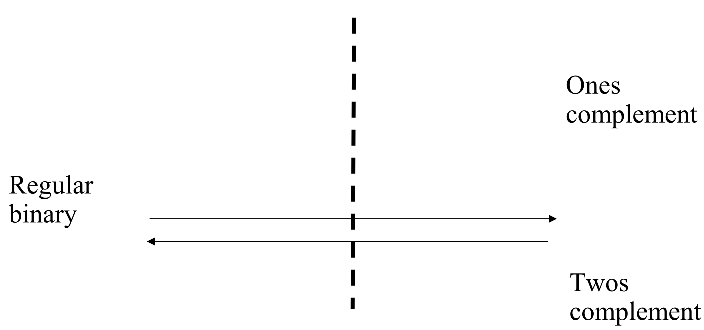
Reminder: Crossing The Boundary Between Regular And Signed Binary



Each time that this boundary is crossed (steps 2 & 4 from the previous slide) apply the rule:

1) Positive numbers pass unchanged
2) Negative numbers must be converted (complemented)
   a. One’s complement: negate the negative number
   b. Two’s complement: negate and add one to the result

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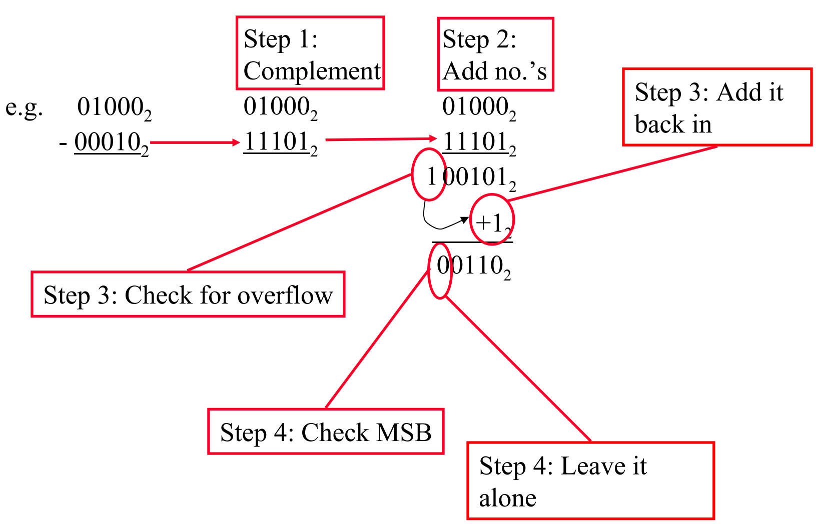
Binary Subtraction Through Ones Complements

1) *Convert from regular binary to a 1's complement representation* (check if it is preceded by a minus sign).
   a. If the number is not preceded by a minus sign, it's positive (leave it alone).
   b. If the number is preceded by a minus sign, the number is negative (complement it by flipping the bits) and remove the minus sign.
2) Add the two binary numbers.
3) Check if there is overflow (a bit is carried out) and if so add it back.
4) *Convert the 1's complement value back to regular binary* (check the value of the MSB).
   a. If the MSB = 0, the number is positive (leave it alone)
   b. If the MSB = 1, the number is negative (complement it by flipping the bits) and precede the number with a minus sign

Binary Subtraction Through Ones Complements



presenting negative and real numbers on the

Overflow: Regular Binary

- Occurs when you don't have enough bits to represent a value (wraps –around to zero)

Binary (1 bit)	Value
0	0
1	1

0 0
: :

Binary (2 bits)	Value
00	0
01	1
10	2
11	3

00 0
: :

Binary (3 bits)	Value
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

000 0
: :

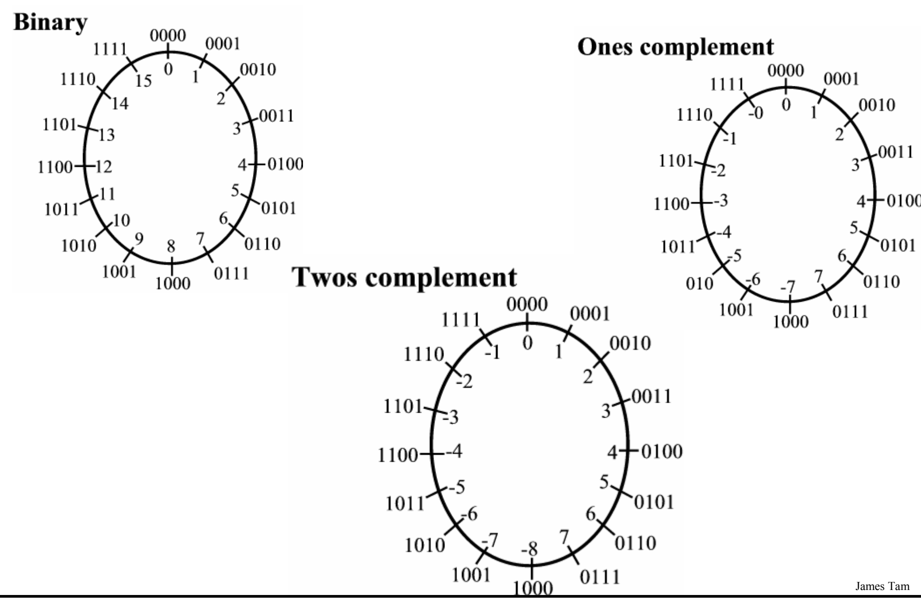
Overflow: Signed

- In all cases it occurs do to a "shortage of bits"
- Subtraction – subtracting two negative numbers results in a positive number.

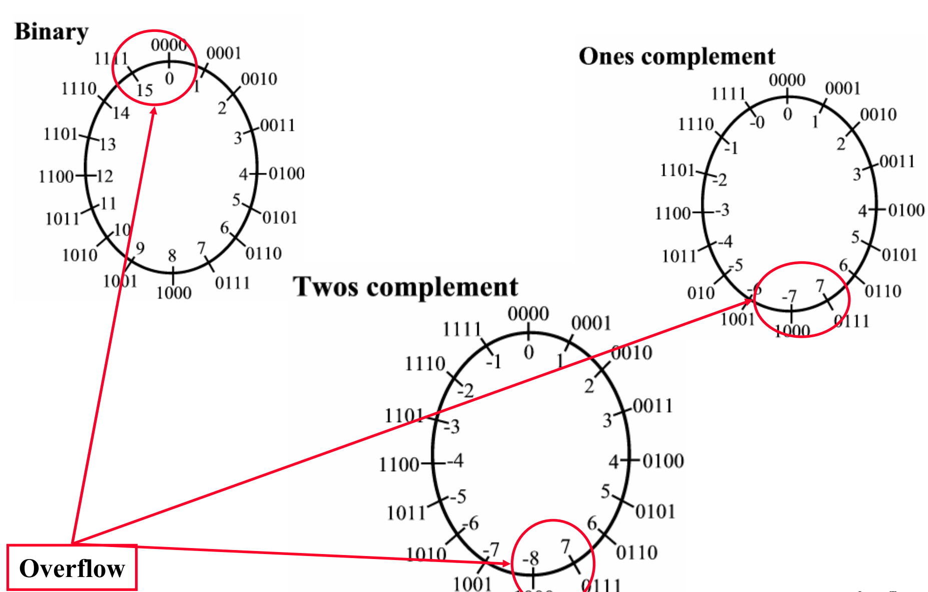
  e.g. - 7
  - 1
  + 7
- Addition – adding two positive numbers results in a negative number.

  e.g. 7
  + 1
  - 8

Summary Diagram Of The 3 Binary Representations



Summary Diagram Of The 3 Binary Representations



Binary Subtraction Through Twos Complements

1) *Convert from regular binary to a 2's complement representation* (check if it's preceded by a minus sign).
   a. If the number is not preceded by a minus sign, it's positive (leave it alone).
   b. If the number is preceded by a minus sign, the number is negative (complement it and discard the minus sign).
      i. Flip the bits.
      ii. Add one to the result.
2) Add the two binary numbers.
3) Check if there is overflow (a bit is carried out) and if so discard it.
4) *Convert the 2's complement value back to regular binary* (check the value of the MSB).
   a. If the MSB = 0, the number is positive (leave it alone).
   b. If the MSB = 1, the number is negative (complement it and precede the number with a negative sign).
      i. Flip the bits.
      ii. Add one to the result.

Binary Subtraction Through Twos Complements

Step 1A: flip bits

Step 1B: add 1

Step 2: Add no's

e.g.	Step 1A: flip bits	Step 1B: add 1	Step 2: Add no's
01000_2	01000_2	01000_2	01000_2
$- \underline{00010}_2$	$\underline{11101}_2$	$\underline{11110}_2$	$\underline{11110}_2$
			$1\ 00110_2$

Step 3: Check for overflow

Binary Subtraction Through Twos Complements

		Step 1A: flip bits	Step 1B: add 1	Step 2: Add no's
e.g.	01000_2	01000_2	01000_2	01000_2
	$- \underline{00010}_2$	$\underline{11101}_2$	$\underline{11110}_2$	$\underline{11110}_2$
				00110_2

Step 3: Discard it

Binary Subtraction Through Twos Complements

		Step 1A: flip bits	Step 1B: add 1	Step 2: Add no's
e.g.	01000_2	01000_2	01000_2	01000_2
	$- \underline{00010}_2$	$\underline{11101}_2$	$\underline{11110}_2$	$\underline{11110}_2$
				00110_2

Step 4: Check MSB

Step 4: Leave it alone

Representing Real Numbers Via Floating Point

- Numbers are represented through a sign bit, a mantissa and an exponent

Sign	Mantissa	Exponent

Examples with 5 digits used to represent the mantissa:
- e.g. One: 123.45 is represented as $12345 * 10^{-2}$
- e.g. Two: 0.12 is represented as $12000 * 10^{-5}$
- e.g. Three: 123456 is represented as $12345 * 10^{1}$

- Using floating point numbers may result in a loss of accuracy!

You Should Now Know

- How negative numbers are represented using ones and twos complement representations.
- How to convert regular binary to values into their ones or twos complement equivalent.
- What is signed overflow and why does it occur.
- How to perform binary subtractions via the negate and add technique.
- How are real numbers represented through floating point representations