

Numerical Representations On The Computer: Negative And Rational Numbers

- How are negative and rational numbers represented on the computer?
- How are subtractions performed by the computer?

James Tam

Subtraction

- In the real world
 $A - B$
- In the computer
 $A - B$

James Tam

Subtraction

- In the real world

$$A - B$$

- In the computer

$$A - B$$

$$A + (-B)$$

Not done this way!

James Tam

Binary Subtraction

- Requires the complementing of a binary number
 - i.e., $A - B$ becomes $A + (-B)$
- The complementing can be performed by representing the negative number as a ones or twos complement value.

James Tam

Complementing Binary Using The Ones Complement Representation

- For positive values there is no difference (no change is needed)
 - e.g., positive seven (The 'A' in the expression $A - B$)
0111 (regular binary)
0111 (Ones complement equivalent)
- For negative values complement the number by negating the binary values: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0).
 - e.g., minus six (The 'B' in the expression $A - B$ becomes $A + (-B)$)
-0110 (regular binary)
1001 (Ones complement equivalent)

James Tam

Complementing Binary Using The Twos Complement Representation

- For positive values there is no difference (no change is needed)
 - e.g., positive seven (The 'A' in the expression $A - B$)
0111 (regular binary)
0111 (Twos complement equivalent)
- For negative values complement the number by negating the number: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0) *and adding one to the result*.
 - e.g., minus six (The 'B' in the expression $A - B$ becomes $A + (-B)$)
-0110 (regular binary)
1010 (Twos complement equivalent)

James Tam

Interpreting The Bit Pattern: Complements

- Recall:
 - Positive values remain unchanged:
 - 0110 is the same value with all three representations.
 - Negative values are converted through complementing:
 - Ones complement: negate the bits
-0110 becomes 1001
 - Twos complement: negate the bits and add one
-0110 becomes 1010
- Problem: the sign must be retained (complements don't use a minus sign).
- Approach:
 - One bit (most significant bit/MSB or the signed bit) is used to indicate the sign of the number.
 - This bit cannot be used to represent the magnitude of the number
 - If the MSB equals 0, then the number is positive
e.g. 0 bbb is a positive number (bbb stands for a binary number)
 - If the MSB equals 1, then the number is negative
e.g. 1 bbb is a negative number (bbb stands for a binary number)

James Tam

Summary Of The Three Binary Representations

| | Positive values are represented with: | Negative values are represented with: |
|-----------------|---|---------------------------------------|
| Regular binary | No explicit symbol is needed (rarely is a plus '+' used) e.g., 100 vs. +100 | A minus '-' sign e.g., -100 |
| Ones complement | The sign bit (MSB) is zero e.g., 011 | The sign bit (MSB) is one e.g., 100 |
| Twos complement | The sign bit (MSB) is zero e.g., 011 | The sign bit (MSB) is one e.g., 100 |

James Tam

Interpreting The Bits: All Representations

| Bit pattern | Regular binary | Ones complement | Twos complement |
|-------------|----------------|-----------------|-----------------|
| 0000 | 0 | 0 | 0 |
| 0001 | 1 | 1 | 1 |
| 0010 | 2 | 2 | 2 |
| 0011 | 3 | 3 | 3 |
| 0100 | 4 | 4 | 4 |
| 0101 | 5 | 5 | 5 |
| 0110 | 6 | 6 | 6 |
| 0111 | 7 | 7 | 7 |
| 1000 | 8 | -7 | -8 |
| 1001 | 9 | -6 | -7 |
| 1010 | 10 | -5 | -6 |
| 1011 | 11 | -4 | -5 |
| 1100 | 12 | -3 | -4 |
| 1101 | 13 | -2 | -3 |
| 1110 | 14 | -1 | -2 |
| 1111 | 15 | -0 | -1 |

James Tam

What You Already Should Know

- How to convert from decimal to binary.
- How to convert from binary to decimal.

James Tam

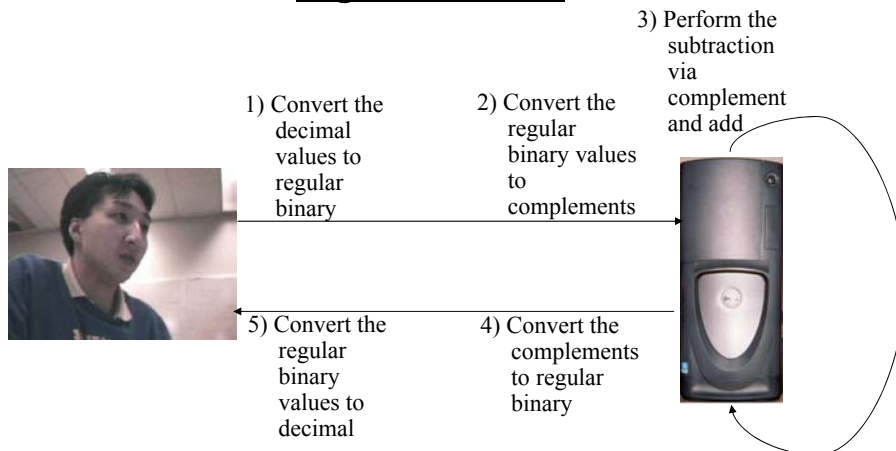
What You Will Learn

- How to subtract numbers with the complement and add technique:

The operation $A - B$ is performed as $A + (-B)$

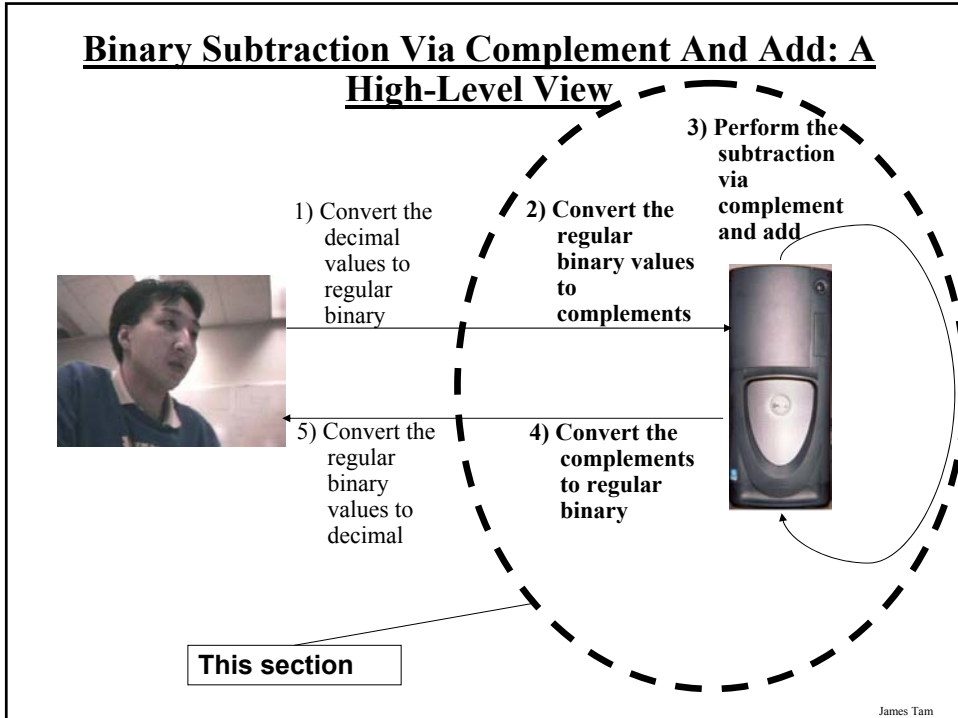
James Tam

Binary Subtraction Via Complement And Add: A High-Level View

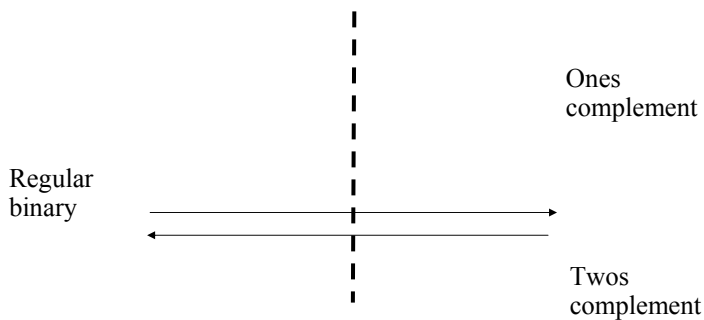


James Tam

Binary Subtraction Via Complement And Add: A High-Level View



Reminder: Crossing The Boundary Between Regular And Signed Binary



Each time that this boundary is crossed (steps 2 & 4 from the previous slide) apply the rule:

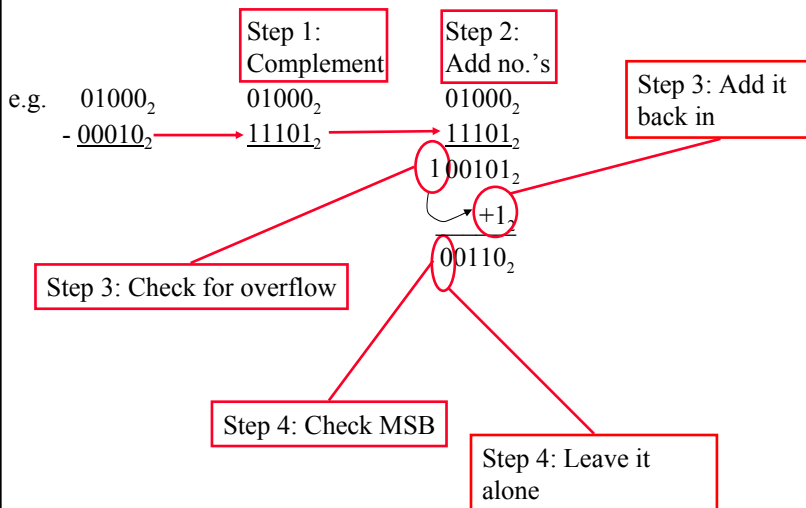
- 1) Positive numbers pass unchanged
- 2) Negative numbers must be converted (complemented)
 - a. One's complement: negate the negative number
 - b. Two's complement: negate and add one to the result

Binary Subtraction Through Ones Complements

- 1) *Convert from regular binary to a 1's complement representation (check if it is preceded by a minus sign).*
 - a. If the number is not preceded by a minus sign, it's positive (leave it alone).
 - b. If the number is preceded by a minus sign, the number is negative (complement it by flipping the bits) and remove the minus sign.
- 2) Add the two binary numbers.
- 3) Check if there is overflow (a bit is carried out) and if so add it back.
- 4) *Convert the 1's complement value back to regular binary (check the value of the MSB).*
 - a. If the MSB = 0, the number is positive (leave it alone)
 - b. If the MSB = 1, the number is negative (complement it by flipping the bits) and precede the number with a minus sign

James Tam

Binary Subtraction Through Ones Complements



James Tam

Overflow: Regular Binary

- Occurs when you don't have enough bits to represent a value (wraps –around to zero)

| Binary (1 bit) | Value |
|----------------|-------|
| 0 | 0 |
| 1 | 1 |

0 0
: :

| Binary (2 bits) | Value |
|-----------------|-------|
| 00 | 0 |
| 01 | 1 |
| 10 | 2 |
| 11 | 3 |

00 0
: :

| Binary (3 bits) | Value |
|-----------------|-------|
| 000 | 0 |
| 001 | 1 |
| 010 | 2 |
| 011 | 3 |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |

000 0
: :

James Tam

Overflow: Signed

- In all cases it occurs do to a “shortage of bits”
- Subtraction – subtracting two negative numbers results in a positive number.

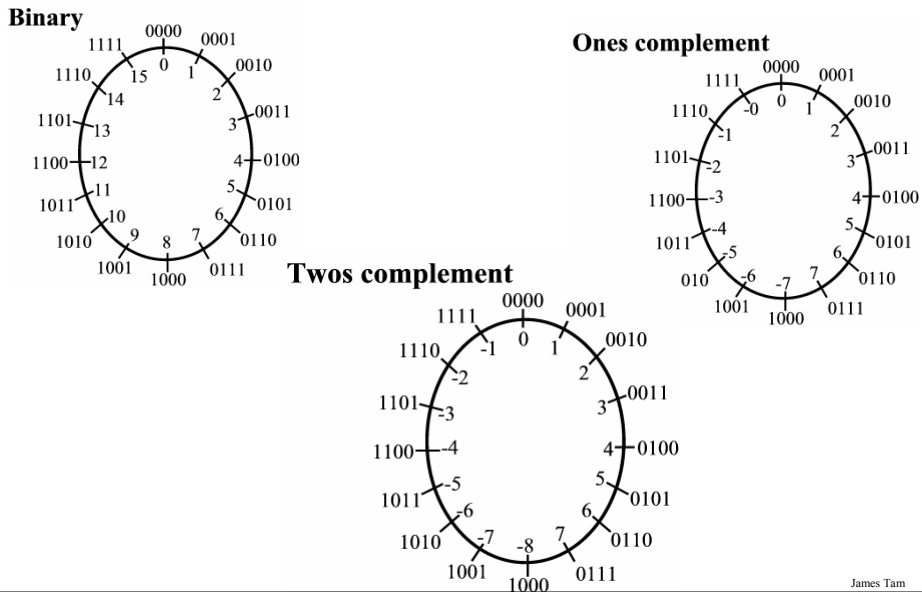
e.g. - 7
- 1
+ 7

- Addition – adding two positive numbers results in a negative number.

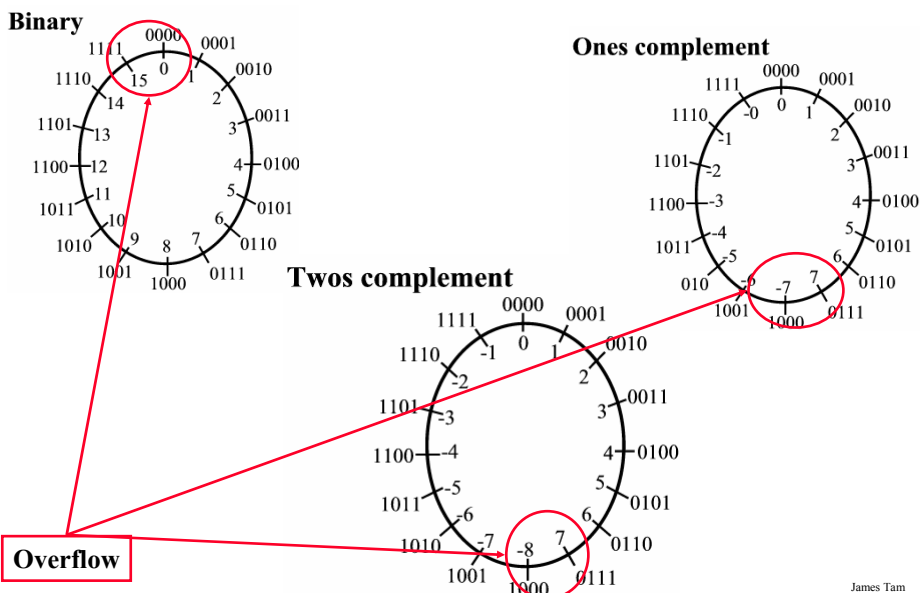
e.g. 7
+ 1
- 8

James Tam

Summary Diagram Of The 3 Binary Representations



Summary Diagram Of The 3 Binary Representations

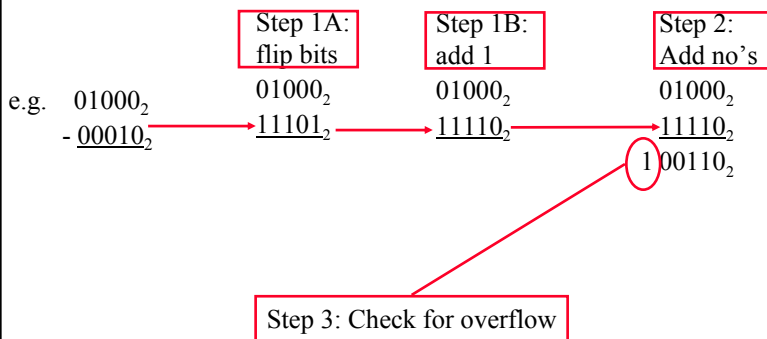


Binary Subtraction Through Twos Complements

- 1) *Convert from regular binary to a 2's complement representation (check if it's preceded by a minus sign).*
 - a. If the number is not preceded by a minus sign, it's positive (leave it alone).
 - b. If the number is preceded by a minus sign, the number is negative (complement it and discard the minus sign).
 - i. Flip the bits.
 - ii. Add one to the result.
- 2) Add the two binary numbers.
- 3) Check if there is overflow (a bit is carried out) and if so discard it.
- 4) *Convert the 2's complement value back to regular binary (check the value of the MSB).*
 - a. If the MSB = 0, the number is positive (leave it alone).
 - b. If the MSB = 1, the number is negative (complement it and precede the number with a negative sign).
 - i. Flip the bits.
 - ii. Add one to the result.

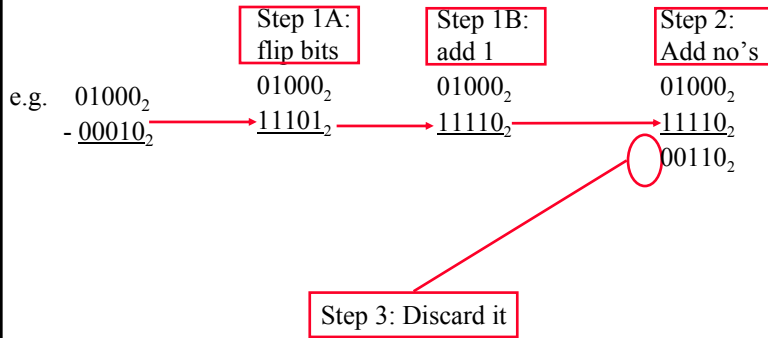
James Tam

Binary Subtraction Through Twos Complements



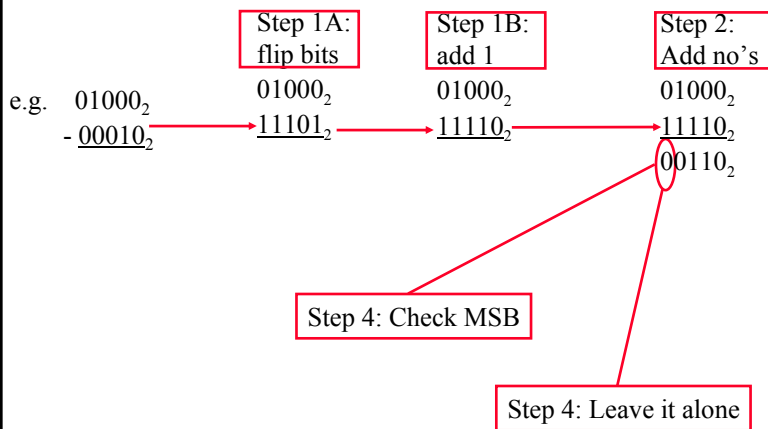
James Tam

Binary Subtraction Through Twos Complements



James Tam

Binary Subtraction Through Twos Complements



James Tam

Representing Real Numbers Via Floating Point

- Numbers are represented through a sign bit, a mantissa and an exponent

Sign

Mantissa

Exponent

Examples with 5 digits used to represent the mantissa:

- e.g. One: 123.45 is represented as $12345 * 10^{-2}$
 - e.g. Two: 0.12 is represented as $12000 * 10^{-5}$
 - e.g. Three: 123456 is represented as $12345 * 10^1$
- Using floating point numbers may result in a loss of accuracy!

James Tam

You Should Now Know

- How negative numbers are represented using ones and twos complement representations.
- How to convert regular binary to values into their ones or twos complement equivalent.
- What is signed overflow and why does it occur.
- How to perform binary subtractions via the negate and add technique.
- How are real numbers represented through floating point representations

James Tam