

Numerical Representations On The Computer: Negative And Rational Numbers

- How are negative and rational numbers represented on the computer?
- How are subtractions performed by the computer?

James Tam

Subtraction

- In the real world
 $A - B$
- In the computer
 $A - B$

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Subtraction

- In the real world

$$A - B$$

- In the computer

~~$$A - B$$~~

~~$$A + (-B)$$~~

Not done this way!

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Binary Subtraction

- Requires the complementing of a binary number
 - i.e., $A - B$ becomes $A + (-B)$
- The complementing can be performed by representing the negative number as a ones or twos complement value.

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Complementing Binary Using The Ones Complement Representation

- For positive values there is no difference (no change is needed)
 - e.g., positive seven (The 'A' in the expression $A - B$)
0111 (regular binary)
0111 (1's complement equivalent)
- For negative values complement the number by negating the binary values: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0).
 - e.g., minus six (The 'B' in the expression $A - B$ becomes $A + (-B)$)
-0110 (regular binary)
1001 (1's complement equivalent)

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Complementing Binary Using The Twos Complement Representation

- For positive values there is no difference (no change is needed)
 - e.g., positive seven (The 'A' in the expression $A - B$)
0111 (regular binary)
0111 (2's complement equivalent)
- For negative values complement the number by negating the number: reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0) *and adding one to the result*.
 - e.g., minus six (The 'B' in the expression $A - B$ becomes $A + (-B)$)
-0110 (regular binary)
1010 (2's complement equivalent)

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Interpreting The Bit Pattern: Complements

- Recall:
 - Positive values remain unchanged:
 - 0110 is the same value with all three representations.
 - Negative values are converted through complementing:
 - Ones complement: negate the bits
-0110 becomes 1001
 - Twos complement: negate the bits and add one
-0110 becomes 1010
- Problem: the sign must be retained (complements don't use a minus sign).
- Approach:
 - One bit (most significant bit/MSB or the signed bit) is used to indicate the sign of the number.
 - This bit cannot be used to represent the magnitude of the number
 - If the MSB equals 0, then the number is positive
e.g. 0 bbb is a positive number (bbb stands for a binary number)
 - If the MSB equals 1, then the number is negative
e.g. 1 bbb is a negative number (bbb stands for a binary number)

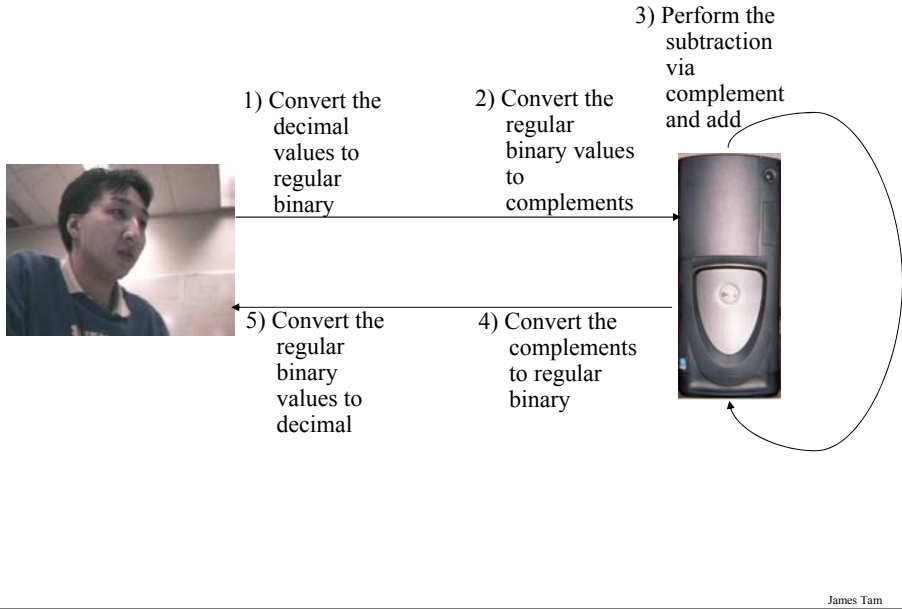
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Interpreting The Bits: All Representations

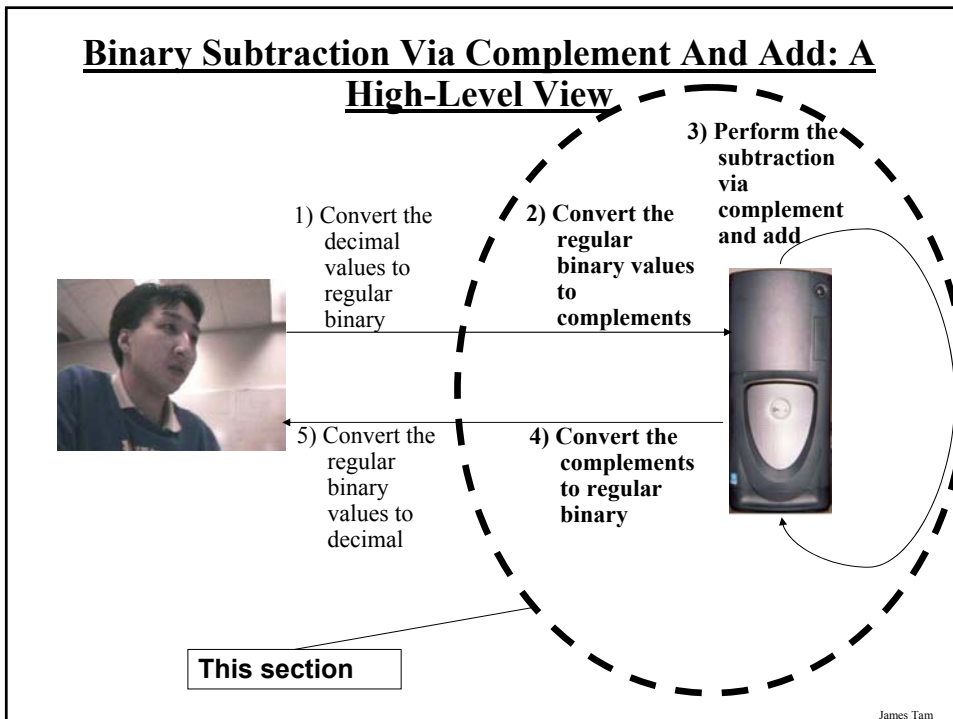
Bit pattern	Regular binary	Ones complement	Twos complement
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

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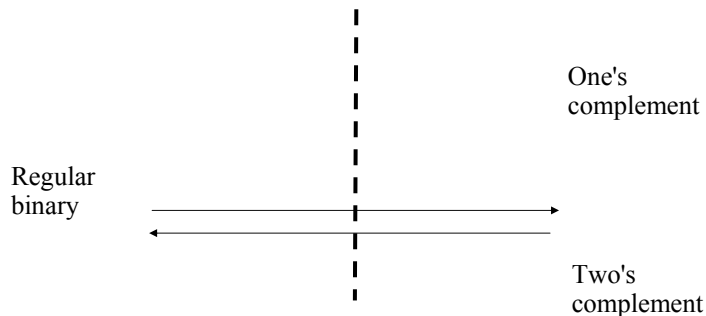
Binary Subtraction Via Complement And Add: A High-Level View



Binary Subtraction Via Complement And Add: A High-Level View



Crossing The Boundary Between Regular And Signed Binary



Each time that this boundary is crossed (steps 2 & 4 from the previous slide) apply the rule:

- 1) Positive numbers pass unchanged
- 2) Negative numbers must be converted (complemented)
 - a. One's complement: negate the negative number
 - b. Two's complement: negate and add one to the result

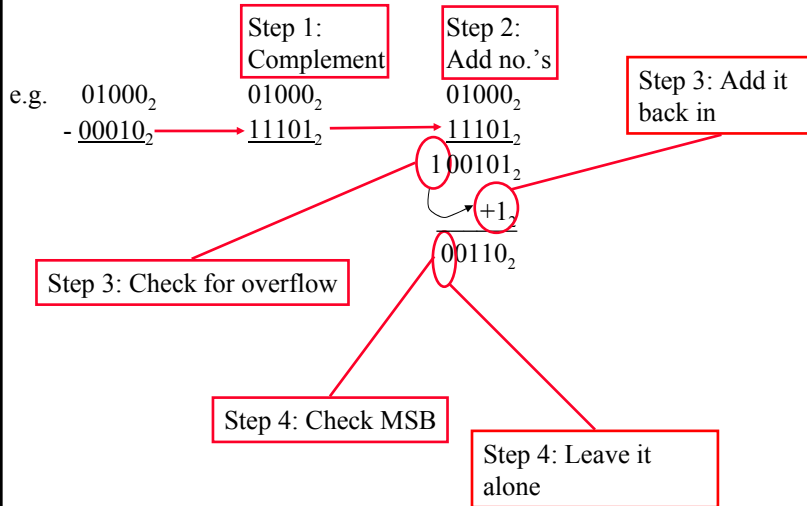
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Binary Subtraction Through Ones Complements

- 1) *Convert from regular binary to a 1's complement representation* (check if it is preceded by a minus sign).
 - a. If the number is not preceded by a minus sign, it's positive (leave it alone).
 - b. If the number is preceded by a minus sign, the number is negative (complement it by flipping the bits) and remove the minus sign.
- 2) Add the two binary numbers.
- 3) Check if there is overflow (a bit is carried out) and if so add it back.
- 4) *Convert the 1's complement value back to regular binary* (check the value of the MSB).
 - a. If the MSB = 0, the number is positive (leave it alone)
 - b. If the MSB = 1, the number is negative (complement it by flipping the bits) and precede the number with a minus sign

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Binary Subtraction Through 1's Complements



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Overflow: Regular Binary

- Occurs when you don't have enough bits to represent a value (wraps –around to zero)

Binary (1 bit)	Value
0	0
1	1

0 0
 : :

Binary (2 bits)	Value
00	0
01	1
10	2
11	3

00 0
 : :

Binary (3 bits)	Value
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

000 0
 : :

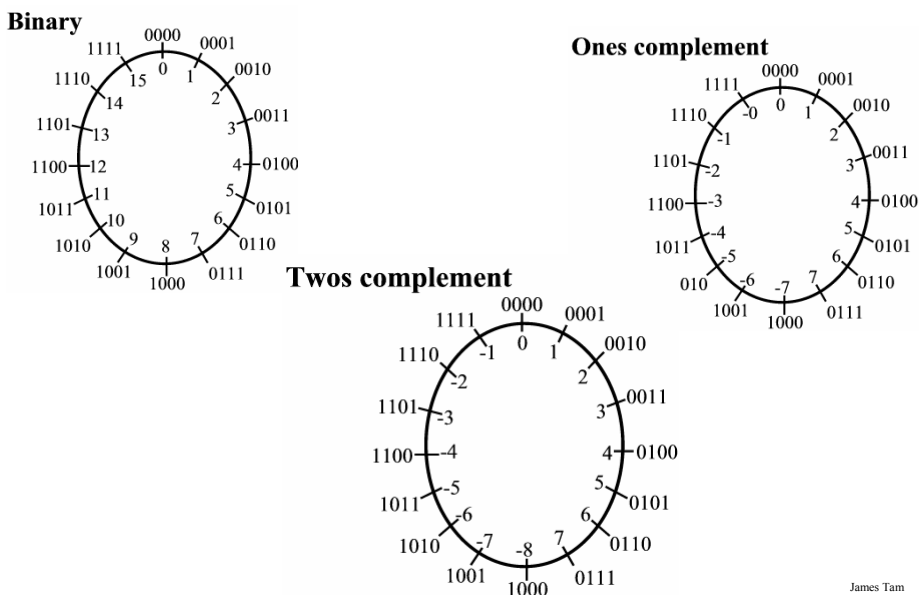
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Overflow: Signed

- In all cases it occurs do to a “shortage of bits”
- Subtraction – subtracting two negative numbers results in a positive number.
 e.g. -7
 $- \frac{1}{7}$
 $+ 7$
- Addition – adding two positive numbers results in a negative number.
 e.g. 7
 $+ \frac{1}{8}$
 $- 8$

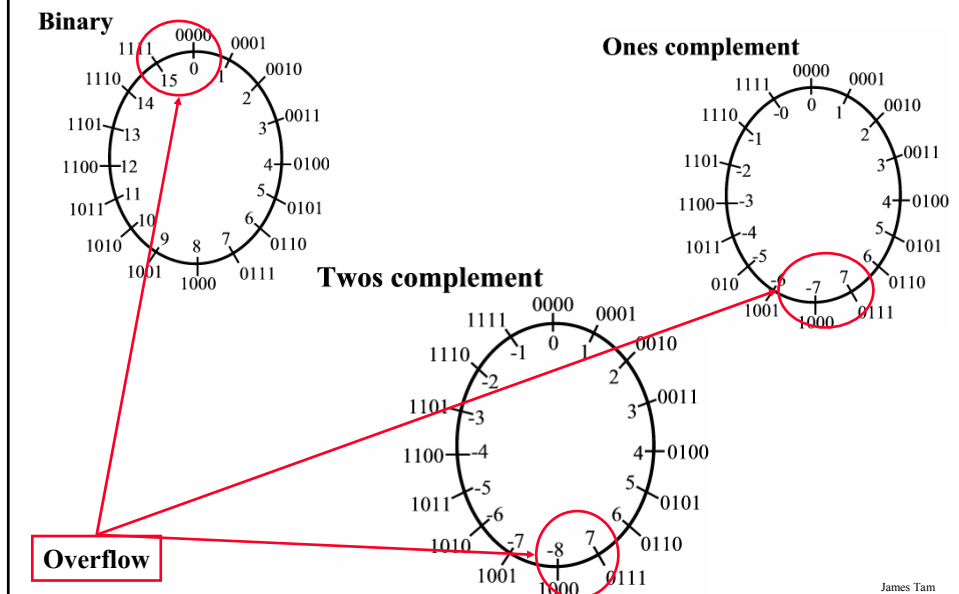
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Summary Diagram Of The 3 Binary Representations



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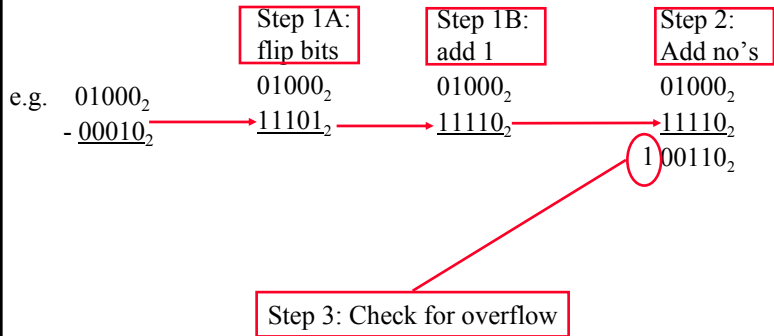
Summary Diagram Of The 3 Binary Representations



Binary Subtraction Through Two's Complements

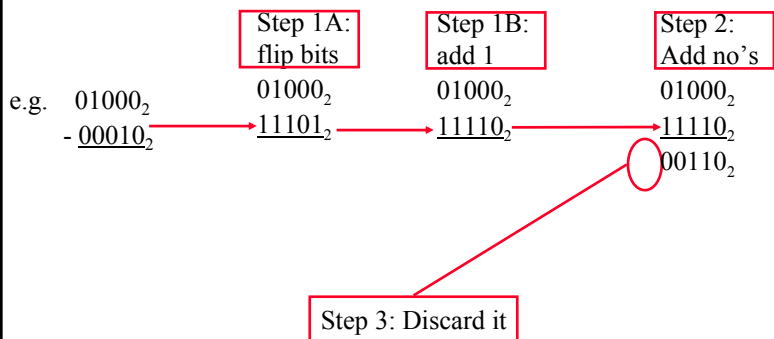
- 1) Convert from regular binary to a 2's complement representation (check if it's preceded by a minus sign).
 - a. If the number is not preceded by a minus sign, it's positive (leave it alone).
 - b. If the number is preceded by a minus sign, the number is negative (complement it and discard the minus sign).
 - i. Flip the bits.
 - ii. Add one to the result.
- 2) Add the two binary numbers.
- 3) Check if there is overflow (a bit is carried out) and if so discard it.
- 4) Convert the 2's complement value back to regular binary (check the value of the MSB).
 - a. If the MSB = 0, the number is positive (leave it alone).
 - b. If the MSB = 1, the number is negative (complement it and precede the number with a negative sign).
 - i. Flip the bits.
 - ii. Add one to the result.

Binary Subtraction Through 2's Complements



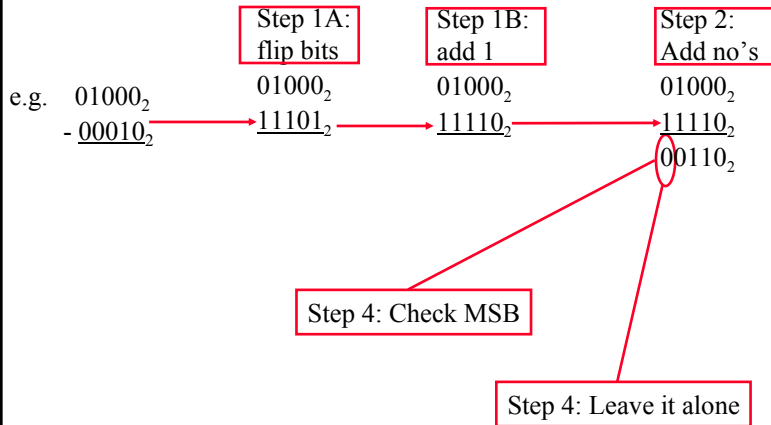
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Binary Subtraction Through 2's Complements



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Binary Subtraction Through 2's Complements



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Representing Real Numbers Via Floating Point

- Numbers are represented through a sign bit, a mantissa and an exponent

Sign	Mantissa	Exponent
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Examples with 5 digits used to represent the mantissa:

- e.g. One: 123.45 is represented as $12345 * 10^{-2}$
 - e.g. Two: 0.12 is represented as $12000 * 10^{-5}$
 - e.g. Three: 123456 is represented as $12345 * 10^1$
- Using floating point numbers may result in a loss of accuracy!

James Tam

You Should Now Know

- How negative numbers are represented using ones and twos complement representations.
- How to convert regular binary to values into their ones or twos complement equivalent.
- What is signed overflow and why does it occur.
- How to perform binary subtractions via the negate and add technique.
- How are real numbers represented through floating point representations