# Beyond Base 10: Non-decimal Based Number Systems

- **What is the decimal based number system?**
- **How do other number systems work (binary, octal and hex)**
- **How to convert to and from non-decimal number systems to decimal**
- **Binary math**

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## What Is Decimal?

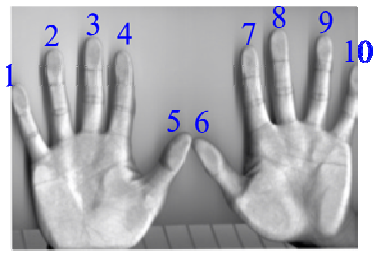
Base 10

- 10 unique symbols are used to represent values

| |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| : |

The largest decimal value that can be represented by a single decimal digit is 9 = base(10) - 1

The number of digits is based on…the number of digits

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## Binary

Base two

Employs two unique symbols (0 and 1)

Largest decimal value that can be represented by 1 binary digit = 1 = base(2) - 1



## Table Of Binary Values

| Decimal value | Binary value | Decimal value | Binary value |
|---|---|---|---|
| 0 | 0000 | 8 | 1000 |
| 1 | 0001 | 9 | 1001 |
| 2 | 0010 | 10 | 1010 |
| 3 | 0011 | 11 | 1011 |
| 4 | 0100 | 12 | 1100 |
| 5 | 0101 | 13 | 1101 |
| 6 | 0110 | 14 | 1110 |
| 7 | 0111 | 15 | 1111 |

## Why Bother With Binary?

Representing information

- ASCII (American Standard Code for Information Interchange)
- Unicode

It's the language of the computer



## Representing Information: ASCII

| ASCII | Decimal | Binary |
|---|---|---|
| Invisible (control characters) | 0 – 31 | 00000000 – 00011111 |
| Punctuation, mathematical operations | 32 – 47 | 00100000 – 00101111 |
| Characters 0 - 9 | 48 - 57 | 00110000 – 00111001 |
| Comparators and other miscellaneous characters : ; ? @ | 58 – 64 | 00111010 – 01000000 |
| Alphabetic (upper case A - Z) | 65 - 90 | 01000001 – 01011010 |
| More miscellaneous characters [ \ ] ^ _ ' | 91 – 96 | 01011011 – 01100000 |
| Alphabetic (lower case a - z) | 97 – 122 | 01100001 – 01111010 |
| More miscellaneous characters { \| } ~ DEL | 123 – 127 | 01111011 - 01111111 |

## Representing Information: ASCII (2)

Uses 7 bits to represent characters

Max number of possibilities = $2^7$ = 128 characters that can be represented

e.g., 'A' is 65 in decimal or 01000001in binary. In memory it looks like this:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |



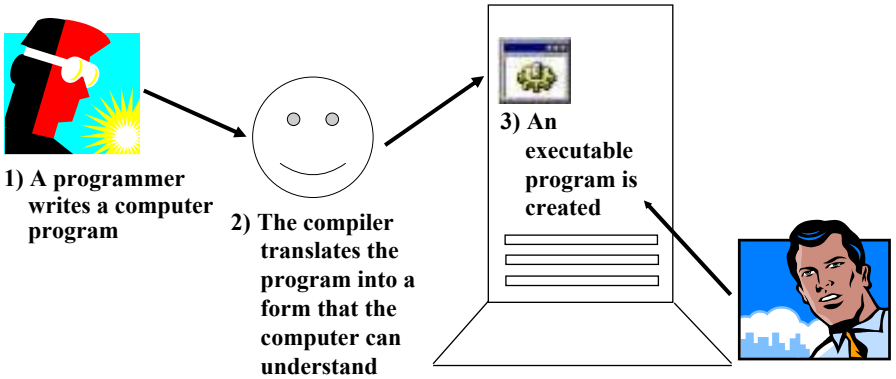
## Representing Information: Unicode

Uses 16 bits (or more) to represent information

Max number of possibilities = $2^{16}$ = 65536 characters that can be represented (more if more bits are used)

## Computer Programs

Binary is the language of the computer



**1) A programmer writes a computer program**

**2) The compiler translates the program into a form that the computer can understand**

**3) An executable program is created**

**4) Anybody who has this executable installed on their computer can then run (use) it.**

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## Octal

Base eight

Employs eight unique symbols (0 - 7)

Largest decimal value that can be represented by 1 octal digit = 7 = base(8) - 1

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## Table Of Octal Values

| Decimal value | Octal value | Decimal value | Octal value |
|---|---|---|---|
| 0 | 0 | 8 | 10 |
| 1 | 1 | 9 | 11 |
| 2 | 2 | 10 | 12 |
| 3 | 3 | 11 | 13 |
| 4 | 4 | 12 | 14 |
| 5 | 5 | 13 | 15 |
| 6 | 6 | 14 | 16 |
| 7 | 7 | 15 | 17 |



## Why Octal?

1001 0100 1100 1100?

1001 0100 1100 0100?

1001 0100 1100 0011?

## Why Octal? (2)

| Machine language | Octal value |
|---|---|
| 1010111000000 | 012700 |
| 1001010000101 | 011205 |



## Hexadecimal (Hex)

Base sixteen

Employs sixteen unique symbols (0 – 9, followed by A - F)

Largest decimal value that can be represented by 1 hex digit = 15

## Table of Hex Values

| Decimal value | Hexadecimal value | Decimal value | Hexadecimal value |
|---|---|---|---|
| 0 | 0 | 9 | 9 |
| 1 | 1 | 10 | A |
| 2 | 2 | 11 | B |
| 3 | 3 | 12 | C |
| 4 | 4 | 13 | D |
| 5 | 5 | 14 | E |
| 6 | 6 | 15 | F |
| 7 | 7 | 16 | 10 |
| 8 | 8 | 17 | 11 |



## Why Hexadecimal?

1001 0100 1000 0000 1100 0100 0110 1010?

Or

1001 0100 1000 0000 1100 0100 0110 1011?

## Why Hexadecimal? (2)

| Machine language | Hexadecimal value |
|---|---|
| 1010011000001 | 14C1 |
| 110000011100000 | 60E0 |

Example from 68000 Family Assembly Language by Clements A.



## Summary (Decimal, Binary, Octal, Hex)

| Decimal | Binary | Octal | Hex | Decimal | Binary | Octal | Hex |
|---|---|---|---|---|---|---|---|
| 0 | 0000 | 0 | 0 | 8 | 1000 | 10 | 8 |
| 1 | 0001 | 1 | 1 | 9 | 1001 | 11 | 9 |
| 2 | 0010 | 2 | 2 | 10 | 1010 | 12 | A |
| 3 | 0011 | 3 | 3 | 11 | 1011 | 13 | B |
| 4 | 0100 | 4 | 4 | 12 | 1100 | 14 | C |
| 5 | 0101 | 5 | 5 | 13 | 1101 | 15 | D |
| 6 | 0110 | 6 | 6 | 14 | 1110 | 16 | E |
| 7 | 0111 | 7 | 7 | 15 | 1111 | 17 | F |

## Arbitrary Number Bases

Base N

Employs N unique symbols

Largest decimal value that can be represented by 1 digit = Base (N) - 1



## Converting Between Different Number Systems

Binary to/from octal

Binary to/from hexadecimal

Octal to/from hexadecimal

Decimal to any base

Any base to decimal

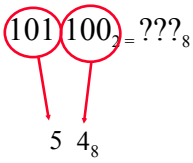
## Binary To Octal

3 binary digits equals one octal digit (remember $2^3=8$)

Form groups of three starting at the decimal

- For the integer portion start grouping at the decimal and go left
- For the fractional portion start grouping at the decimal and go right

e.g. $101\,100_2 = ???_8$



$5\ 4_8$



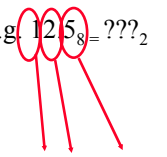
## Octal To Binary

1 octal digit equals = 3 binary digits

Split into groups of three starting at the decimal

- For the integer portion start splitting at the decimal and go left
- For the fractional portion start splitting at the decimal and go right

e.g. $12.5_8 = ???_2$



$001\,010\,.\,101_2$

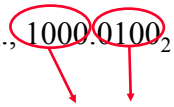
## Binary To Hexadecimal

4 binary digits equals one hexadecimal digit (remember $2^4=16$)

Form groups of four at the decimal

- For the integer portion start grouping at the decimal and go left
- For the fractional portion start grouping at the decimal and go right

e.g., $1000.0100_2 = ???_{16}$



$8 . 4_{16}$

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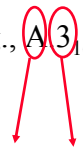
## Hexadecimal To Binary

1 hex digit equals = 4 binary digits

Split into groups of four starting at the decimal

- For the integer portion start splitting at the decimal and go left
- For the fractional portion start splitting at the decimal and go right

e.g., $A3_{16} = ???_2$



$1010.0011_2$

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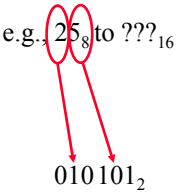
# Octal To Hexadecimal

Convert to binary first!

e.g., $25_8$ to $???_{16}$





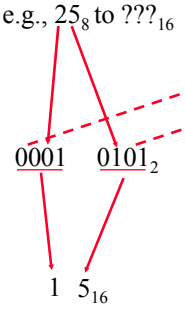

# Octal To Hexadecimal

Convert to binary first!

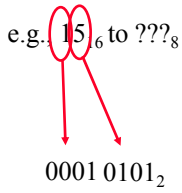
e.g., $25_8$ to $???_{16}$

**Add any leading zeros that are needed (in this case two).**

**Regroup in groups of 4**

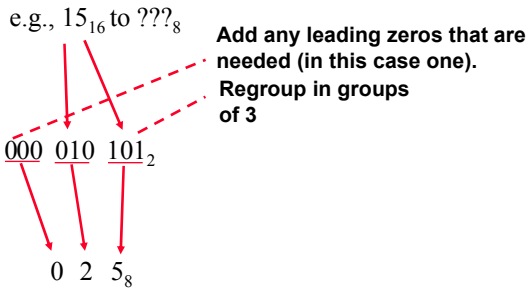


# Hexadecimal To Octal






# Hexadecimal To Octal



## Decimal To Any Base

Split up the integer and the fractional portions

1) For the integer portion:
   a. Divide the integer portion of the decimal number by the target base.
   b. The remainder becomes the first integer digit of the number (immediately left of the decimal) in the target base.
   c. The quotient becomes the new integer value.
   d. Divide the new integer value by the target base.
   e. The new remainder becomes the second integer digit of the converted number (second digit to the left of the decimal).
   f. Continue dividing until the quotient is less than the target base and this quotient becomes the last integer digit of the converted number.

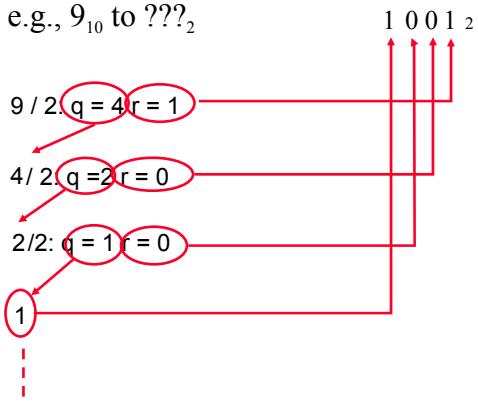


## Decimal To Any Base (2)

2) For the fractional portion:
   a. Multiply by the target base.
   b. The integer portion (if any) of the product becomes the first rational digit of the converted number (first digit to the right of the decimal).
   c. The non-rational portion of the product is then multiplied by the target base.
   d. The integer portion (if any) of the new product becomes the second rational digit of the converted number (second digit to the right of the decimal).
   e. Keep multiplying by the target base until either the resulting product equals zero or you have the desired number of places of precision.

# Decimal To Any Base (2)

e.g., $9_{10}$ to $???_2$




**Stop dividing! (quotient less than target base)**



# Any Base To Decimal

Multiply each digit by the base raised to some exponent[1] and sum the resulting products.

| | 3 | 2 | 1 | 0 | -1 | -2 | -3 | ← Position of digits |
|---|---|---|---|---|---|---|---|---|
| i.e. | d7 | d6 | d5 | d4. | d3 | d2 | $d1_b$ | ← Number to be converted |

Base = b

Value in decimal = (digit7*$b^3$) + (digit6*$b^2$) + (digit5*$b^1$) + (digit4*$b^0$) + (digit3*$b^{-1}$) + (digit2*$b^{-2}$) + (digit1*$b^{-3}$)

1 The value of this exponent will be determined by the position of the digit.

## Any Base To Decimal (2)

e.g., $12_8$ to $???_{10}$

1 0 ← Position of the digits

1 2 ← Number to be converted

Base = 8

Value in decimal = $(1*8^1) + (2*8^0)$

$= (1*8) + (2*1)$

$= 8 + 2$

$= 10_{10}$



## Addition In Binary: Five Cases

Case 1: sum = 0, no carry out

```
  0
+ 0
  0
```

Case 2: sum = 1, no carry out

```
  0
+ 1
  1
```

Case 3: sum = 1, no carry out

```
  1
+ 0
  1
```

Case 4: sum 0, carry out = 1

```
 1
  1
+ 1
1 0
```

1 + 1 = 2 (in decimal)
= 10 (in binary)

# Addition In Binary: Five Cases (2)

Case 5: Sum = 1, Carry out = 1

```
  1
    1
    1
+   1
  1 1
```

1 + 1 + 1 = 3 (in decimal)
= 11 (in binary)



# Subtraction In Binary (4 cases)

Case 1:

```
  0
- 0
  0
```

Case 2:

```
  1
- 1
  0
```

Case 3:

```
  1
- 0
  1
```

Case 4:

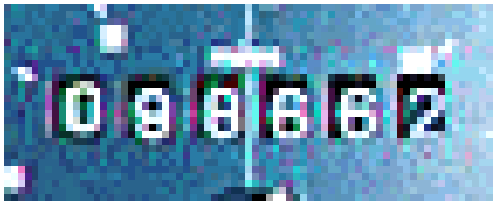
```
  0 2
  1 0
- 1
  1
```

The amount that you borrow equals the base

- Decimal: Borrow 10
- Binary: Borrow 2

## Overflow: A Real World Example

You can only represent a finite number of values





## Overflow: Binary

Occurs when you don't have enough bits to represent a value ("wraps around" to zero)

| Binary (1 bit) | Value |
|---|---|
| 0 | 0 |
| 1 | 1 |

0 0

1 1

: :

| Binary (2 bits) | Value |
|---|---|
| 00 | 0 |
| 01 | 1 |
| 10 | 2 |
| 11 | 3 |

00 0

01 1

10 2

11 3

: :

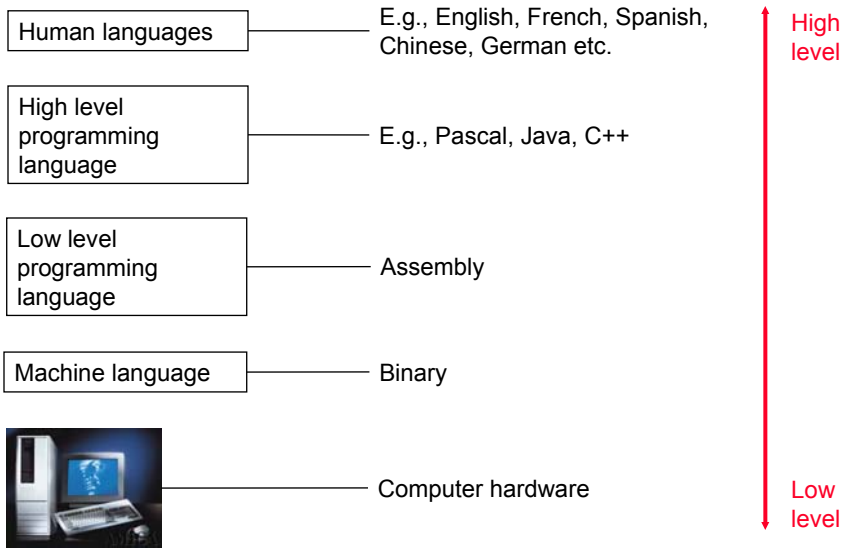
| Binary (3 bits) | Value |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 2 |
| 011 | 3 |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |

000 0

001 1

: :

## High Vs. Low Level






## You Should Now Know

- What is meant by a number base.
- How binary, octal and hex based number systems work and what role they play in the computer.
- How to/from convert between non-decimal based number systems and decimal.
- How to perform simple binary math (addition and subtraction).
- What is overflow, why does it occur and when does it occur.
- What is the difference between a high and low level programming language.