Numerical Representations On The Computer: Negative And Rational Numbers

- **How are negative and rational numbers represented on the computer?**
- **How are subtractions performed by the computer?**

Subtraction

In the real world

A - B

In the computer

A - B

Subtraction

In the real world

A - B

In the computer

~~A - B~~

A + (-B)

Not done this way!

Representing Negative Numbers

Real world

- Negative numbers – same as the case of positive numbers but precede the number with a negative sign "-" e.g., -123456.

Computer world

- Negative numbers – employ *signed representations*.

Magnitude Of Signed Representations

All of the digits are used to represent the magnitude of the number e.g., 175_{10}, 1001_2

An explicit minus sign is needed to distinguish positive and negative numbers e.g., 124_{10} vs. -124_{10} or 100_2 vs. -100_2

Signed Binary

One bit (most significant bit/MSB or the signed bit) is used to indicate the sign of the number.

This bit cannot be used to represent the magnitude of the number

If the MSB equals 0, then the number is positive
- e.g. 0 bbb is a positive number (bbb stands for a binary number)

If the MSB equals 1, then the number is negative
- e.g. 1 bbb is a negative number (bbb stands for a binary number)

Types of signed representations
- One's complement
- Two's complement

representing negative and real numbers on the

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Positive

Negative

Types of signed representations
- •One's complement
- •Two's complement

James Tam

Binary Subtraction

Requires the negation of a binary number

i.e., A – B becomes A + (-B)

The negation can be performed using the One's complement representation or the Two's complement representation

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Negating Regular Binary Using The One's Complement Representation

For positive values there is no difference (no negation is needed)

e.g., positive seven

0111 (regular binary)
0111 (1's complement equivalent)

For negative values negate the number by reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0).

e.g., minus six

-0110 (regular binary)
1001 (1's complement equivalent)

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Negating Regular Binary Using The Two's Complement Representation

For positive values there is no difference (no negation is needed)

e.g., positive seven

0111 (regular binary)
0111 (2's complement equivalent)

For negative values negate the number by reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0) *and add one to the result*.

e.g., minus six

-0110 (regular binary)
1010 (2's complement equivalent)

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Interpreting The Pattern Of Bits For Numbers

Bit pattern	Regular binary	1's complement	2's complement
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

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Overflow: Regular Binary

Occurs when you don't have enough bits to represent a value (wraps –around to zero)

Binary (1 bit)	Value
0	0
1	1

0 0
: :

Binary (2 bits)	Value
00	0
01	1
10	2
11	3

00 0
: :

Binary (3 bits)	Value
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

000 0
: :

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Overflow: Signed

In all cases it occurs do to a "shortage of bits"

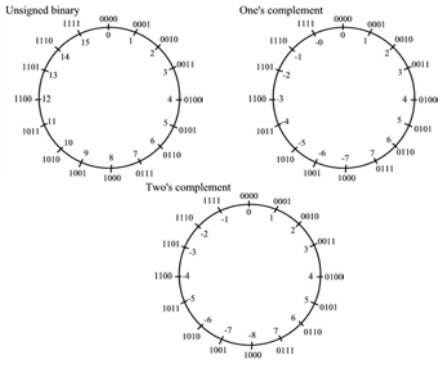
Subtraction – subtracting two negative numbers results in a positive number.

e.g. - 7
- 1
+ 7

Addition – adding two positive numbers results in a negative number.

e.g. 7
+ 1
- 8

Summary Diagram Of The 3 Binary Representations



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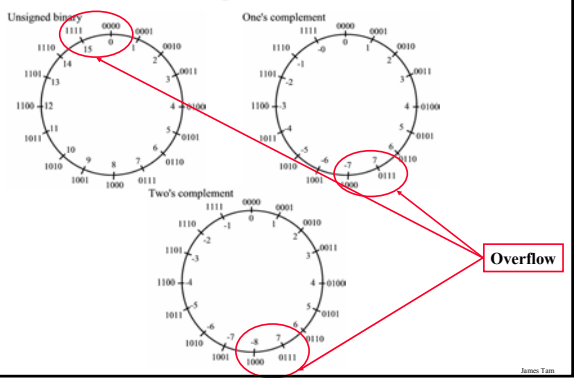
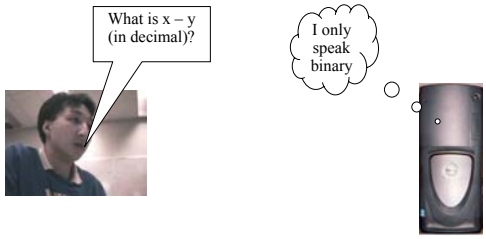


Binary Subtraction Via Negate And Add: A High-Level View



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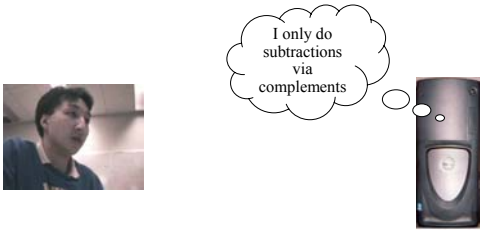

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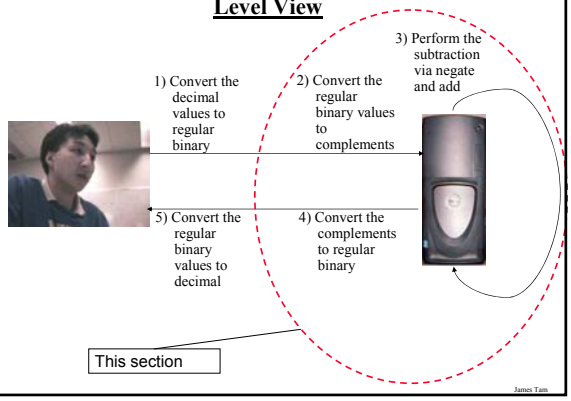

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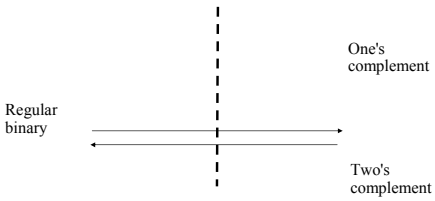
Crossing The Boundary Between Regular And Signed Binary



Each time that this boundary is crossed (steps 2 & 4) apply the rule:

1) Positive numbers pass unchanged
2) Negative numbers must be converted (negated)

James Tam

Binary Subtraction Through One's Complements

1) *Convert from regular binary to a 1's complement representation* (check if it is preceded by a minus sign)
   - If the number is not preceded by a minus sign, it's positive (leave it alone).
   - If the number is preceded by a minus sign, the number is negative (negate it by flipping the bits) and remove the minus sign.
2) Add the two binary numbers
3) Check if there is overflow (a bit is carried out) and if so add it back.
4) *Convert the 1's complement value back to regular binary* (check the value of the MSB)
   - If the MSB = 0, the number is positive (leave it alone)
   - If the MSB = 1, the number is negative (negate it by flipping the bits) and precede the number with a minus sign

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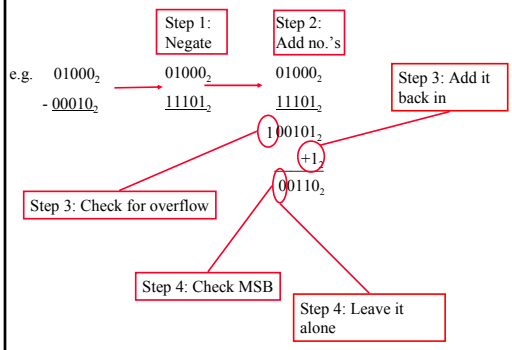
Binary Subtraction Through 1's Complements



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Binary Subtraction Through Two's Complements

1) *Convert from regular binary to a 2's complement representation* (check if it is preceded by a minus sign)
   - If the number is not preceded by a minus sign, it's positive (leave it alone).
   - If the number is preceded by a minus sign, the number is negative (negate it and discard the minus sign).
     a) Flip the bits.
     b) Add one to the result.
2) Add the two binary numbers
3) Check if there is overflow (a bit is carried out) and if so discard it.
4) *Convert the 2's complement value back to regular binary* (check the value of the MSB)
   - If the MSB = 0, the number is positive (leave it alone)
   - If the MSB = 1, the number is negative (negate it and precede the number with a negative sign)
     a) Flip the bits.
     b) Add one to the result.

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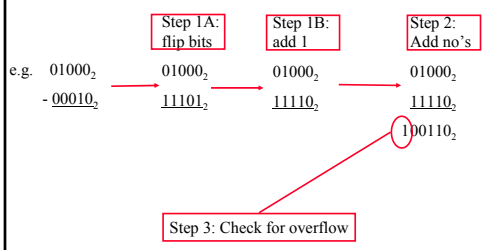
Binary Subtraction Through 2's Complements



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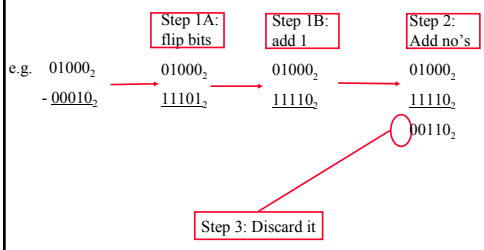
Binary Subtraction Through 2's Complements



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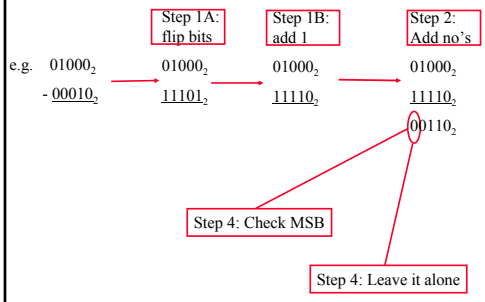
Binary Subtraction Through 2's Complements



Representing Real Numbers Via Floating Point

Numbers are represented through a sign bit, a mantissa and an exponent

Sign	Mantissa	Exponent

Examples with 5 digits used to represent the mantissa:

- e.g. One: 123.45 is represented as $12345 * 10^{-2}$
- e.g. Two: 0.12 is represented as $12000 * 10^{-5}$
- e.g. Three: 123456 is represented as $12345 * 10^{1}$

Floating point numbers may result in a loss of accuracy!

You Should Now Know

- How negative numbers are represented using 1's and 2's complements.
- How to convert regular binary to values into their 1's or 2's complement equivalent.
- What a signed overflow and why it occurs.
- How to perform binary subtractions via the negate and add technique.
- How are real numbers represented through floating point representations

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