

Beyond Base 10: Non-decimal Based Number Systems

- What is the decimal based number system?
- How do other number systems work (binary, octal and hex)
- How to convert to and from non-decimal number systems to decimal
- Binary math

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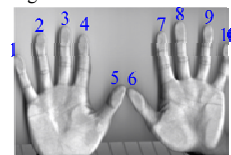
What Is Decimal?

Base 10

- 10 unique symbols are used to represent values

0
1
2
3
4
5
6
7
8
9
10
:

The number of digits is based on...the number of digits



The largest decimal value that can be represented by a single decimal digit is 9 = base(10) - 1

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Binary

Base two

Employs two unique symbols (0 and 1)

Largest decimal value that can be represented by 1 binary digit = 1 = base(2) - 1

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Table Of Binary Values

Decimal value	Binary value	Decimal value	Binary value
0	0000	8	1000
1	0001	9	1001
2	0010	10	1010
3	0011	11	1011
4	0100	12	1100
5	0101	13	1101
6	0110	14	1110
7	0111	15	1111

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Why Bother With Binary?

Representing information

- ASCII (American Standard Code for Information Interchange)
- Unicode

It's the language of the computer

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Representing Information: ASCII

Decimal	Binary	ASCII
0 - 31	00000000 - 00011111	Invisible (control characters)
32 - 47	00100000 - 00101111	Punctuation, mathematical operations
48 - 57	00110000 - 00111001	Characters 0 - 9
58 - 64	00111010 - 01000000	Comparators and other miscellaneous characters ; ; ? @
65 - 90	01000001 - 01011010	Alphabetic (upper case A - Z)
91 - 96	01011011 - 01100000	More miscellaneous characters [\] ^ _ '
97 - 122	01100001 - 01111010	Alphabetic (lower case a - z)
123 - 127	01111011 - 01111111	More miscellaneous characters { } - DEL

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Representing Information: ASCII (2)

Uses 7 bits to represent characters

Max number of possibilities = $2^7 = 128$ characters that can be represented

e.g., 'A' is 65 in decimal or 01000001 in binary. In memory it looks like this:

0	1	0	0	0	0	0	1
---	---	---	---	---	---	---	---

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Representing Information: Unicode

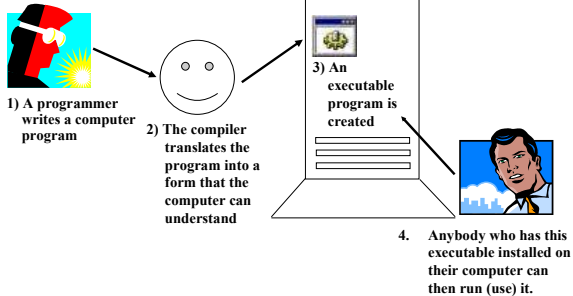
Uses 16 bits (or more) to represent information

Max number of possibilities = $2^{16} = 65536$ characters that can be represented (more if more bits are used)

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Computer Programs

Binary is the language of the computer



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Octal

Base eight

Employs eight unique symbols (0 - 7)

Largest decimal value that can be represented by 1 octal digit = $7 = \text{base}(8) - 1$

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Table Of Octal Values

Decimal value	Octal value	Decimal value	Octal value
0	0	8	10
1	1	9	11
2	2	10	12
3	3	11	13
4	4	12	14
5	5	13	15
6	6	14	16
7	7	15	17

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Uses Of Octal (Assembly Language)

Machine language	Octal value	PDP -11 assembly language
1010111000000	012700	MOV #4, R0
1001010000101	011205	MOV (R2), R5

Example from Introduction to the PDP-11 and its Assembly Language by Frank T.

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Hexadecimal (Hex)

Base sixteen

Employs sixteen unique symbols (0 – 9, followed by A - F)

Largest decimal value that can be represented by 1 hex digit = 15

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Table of Hex Values

Decimal value	Hexadecimal value	Decimal value	Hexadecimal value
0	0	9	9
1	1	10	A
2	2	11	B
3	3	12	C
4	4	13	D
5	5	14	E
6	6	15	F
7	7	16	10
8	8	17	11

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Uses Of Hexadecimal (Assembly Language)

Machine language	Hexadecimal value	680X0 assembly language
1010011000001	14C1	MOV.B D1, (A2)+
110000011100000	60E0	BRA NEXT

Example from 68000 Family Assembly Language by Clements A

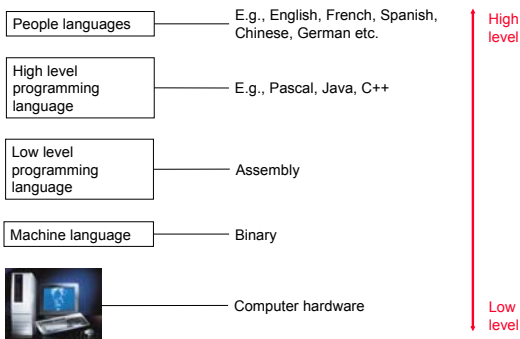
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Summary (Decimal, Binary, Octal, Hex)

Decimal	Binary	Octal	Hex	Decimal	Binary	Octal	Hex
0	0000	0	0	8	1000	10	8
1	0001	1	1	9	1001	11	9
2	0010	2	2	10	1010	12	A
3	0011	3	3	11	1011	13	B
4	0100	4	4	12	1100	14	C
5	0101	5	5	13	1101	15	D
6	0110	6	6	14	1110	16	E
7	0111	7	7	15	1111	17	F

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High Vs. Low Level



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Overflow: A Real World Example

You can only represent a finite number of values



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Overflow: Binary

Occurs when you don't have enough bits to represent a value ("wraps around" to zero)

Binary (1 bit)	Value
0	0
1	1

0 0
1 1
:

Binary (2 bits)	Value
00	0
01	1
10	2
11	3

00 0
01 1
10 2
11 3
:

Binary (3 bits)	Value
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

000 0
001 1

:

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Arbitrary Number Bases

Base N

Employs N unique symbols

Largest decimal value that can be represented by 1 digit = Base (N) - 1

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Converting Between Different Number Systems

Binary to/from octal

Binary to/from hexadecimal

Octal to/from hexadecimal

Decimal to any base

Any base to decimal

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Binary To Octal

3 binary digits equals one octal digit (remember $2^3=8$)

Form groups of three starting at the decimal

- For the integer portion start grouping at the decimal and go left
- For the fractional portion start grouping at the decimal and go right

e.g. $(101100)_2 = ???_8$
5 4_8 .

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Octal To Binary

1 octal digit equals = 3 binary digits

Split into groups of three starting at the decimal

- For the integer portion start splitting at the decimal and go left
- For the fractional portion start splitting at the decimal and go right

e.g. $(25)_8 = ???_2$
001 010 , 101₂

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Binary To Hexadecimal

4 binary digits equals one hexadecimal digit (remember $2^4=16$)

Form groups of four at the decimal

- For the integer portion start grouping at the decimal and go left
- For the fractional portion start grouping at the decimal and go right

e.g. $(10000100)_2 = ???_{16}$
8 . 4_{16}

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Hexadecimal To Binary

1 hex digit equals = 4 binary digits

Split into groups of four starting at the decimal

- For the integer portion start splitting at the decimal and go left
- For the fractional portion start splitting at the decimal and go right

e.g., $A3_{16} = ???_2$

1010.0011_2

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Octal To Hexadecimal

Convert to binary first!

e.g., 25_8 to $??_{16}$

010101_2

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Octal To Hexadecimal

Convert to binary first!

e.g., 25_8 to $??_{16}$

Add any leading zeros that are needed (in this case two).

Regroup in groups of 4

$0001 0101_2$

$1 5_{16}$

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Hexadecimal To Octal

e.g., 15_{16} to $??_8$

$0001 0101_2$

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Hexadecimal To Octal

e.g., 15_{16} to $??_8$

Add any leading zeros that are needed (in this case one).

Regroup in groups of 3

$000 010 101_2$

$0 2 5_8$

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Decimal To Any Base

Split up the integer and the fractional portions

- 1) For the integer portion:
 - a. Divide the integer portion of the decimal number by the target base.
 - b. The remainder becomes the first integer digit of the number in the target base.
 - c. The quotient becomes the new integer value.
 - d. Divide the new integer value by the target base.
 - e. The new remainder becomes the second integer digit of the converted number.
 - f. Continue dividing until the quotient is less than the target base and this quotient becomes the last integer digit of the converted number.

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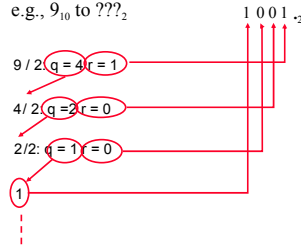
Decimal To Any Base (2)

- 2) For the fractional portion:
- Multiply by the target base.
 - The integer portion (if any) of the product becomes the first rational digit of the converted number.
 - The non-rational portion of the product is then multiplied by the target base.
 - The integer portion (if any) of the new product becomes the second rational digit of the converted number.
 - Keep multiplying by the target base until either the resulting product equals zero or you have the desired number of places of precision.

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Decimal To Any Base (2)

e.g., 9_{10} to $???_2$



Stop dividing! (quotient less than target base)

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Any Base To Decimal

Multiply each digit by the base raised to some exponent, and sum the resulting products.

i.e. $d_7 d_6 d_5 d_4 d_3 d_2 d_1$

Position of digits: 3 2 1 0 -1 -2 -3
 Number to be converted: $d_7 d_6 d_5 d_4 d_3 d_2 d_1$

Base = b

$$\text{Value in decimal} = (\text{digit}7 \cdot b^3) + (\text{digit}6 \cdot b^2) + (\text{digit}5 \cdot b^1) + (\text{digit}4 \cdot b^0) + (\text{digit}3 \cdot b^{-1}) + (\text{digit}2 \cdot b^{-2}) + (\text{digit}1 \cdot b^{-3})$$

¹ The value of this exponent will be determined by the position of the digit.

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Any Base To Decimal (2)

e.g., 12_8 to $???_{10}$

Position of the digits: 1 0
 Number to be converted: 1 2

Base = 8

$$\begin{aligned} \text{Value in decimal} &= (1 \cdot 8^1) + (2 \cdot 8^0) \\ &= (1 \cdot 8) + (2 \cdot 1) \\ &= 8 + 2 \\ &= 10_{10} \end{aligned}$$

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Addition In Binary: Five Cases

Case 1: sum = 0, no carry out

$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array}$$

Case 2: sum = 1, no carry out

$$\begin{array}{r} 0 \\ + 1 \\ \hline 1 \end{array}$$

Case 3: sum = 1, no carry out

$$\begin{array}{r} 1 \\ + 0 \\ \hline 1 \end{array}$$

Case 4: sum 0, carry out = 1

$$\begin{array}{r} 1 \\ + 1 \\ \hline 1 \ 0 \end{array}$$

$1 + 1 = 2$ (in decimal)
 $= 10$ (in binary)

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Addition In Binary: Five Cases (2)

Case 5: Sum = 1, Carry out = 1

$$\begin{array}{r} 1 \\ 1 \\ + 1 \\ \hline 1 \ 1 \end{array}$$

$1 + 1 + 1 = 3$ (in decimal)
 $= 11$ (in binary)

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Subtraction In Binary (4 cases)

Case 1:

$$\begin{array}{r} 0 \\ - 0 \\ \hline 0 \end{array}$$

Case 2:

$$\begin{array}{r} 1 \\ - 1 \\ \hline 0 \end{array}$$

Case 3:

$$\begin{array}{r} 1 \\ - 0 \\ \hline 1 \end{array}$$

Case 4:

$$\begin{array}{r} 0 \quad 2 \\ \swarrow \quad \searrow \\ 1 \quad 1 \\ - 1 \\ \hline 1 \end{array}$$

The amount that you borrow equals the base
•Decimal: Borrow 10
•Binary: Borrow 2

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You Should Now Know

- What is meant by a number base.
- How binary, octal and hex based number systems work and what role they play in the computer.
- What is overflow, why does it occur and when does it occur.
- How to/from convert between non-decimal based number systems and decimal.
- How to perform simple binary math (addition and subtraction).

James Tam