Numerical Representations On The Computer: Negative And Rational Numbers

- **How are negative and rational numbers represented on the computer?**
- **How are subtractions performed by the computer?**

Subtraction

In the real world

A - B

In the computer

A - B

Subtraction

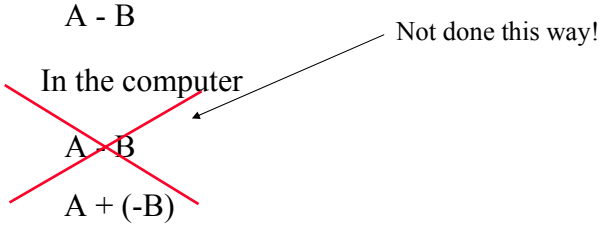
In the real world

A - B

In the computer

A - B

A + (-B)



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Representing Negative Numbers

Real world

- Positive numbers – just use the appropriate type and number of digits e.g., 12345.
- Negative numbers – same as the case of positive numbers but precede the number with a negative sign "-" e.g., -123456.

Computer world

- Positive numbers – use an *unsigned representation* (convert the number to binary e.g., 7 becomes 111)
- Negative numbers – employ *signed representations*.

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Unsigned Representations

All of the digits are used to represent the magnitude of the number e.g., 175

An explicit minus sign is needed to distinguish positive and negative numbers e.g., 124 vs. -124

Decimal is an unsigned representation

The binary that you have already learned about is unsigned

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Signed Binary

One bit (most significant bit/MSB or the signed bit) is used to indicate the sign of the number.

This bit cannot be used to represent the magnitude of the number

If the MSB equals 0 then the number is positive
- e.g. 0 bbb is a positive number (bbb stands for a binary number)

If the MSB equals 1 then the number is negative
- e.g. 1 bbb is a negative number (bbb stands for a binary number)

Types of signed representations
- One's complement
- Two's complement

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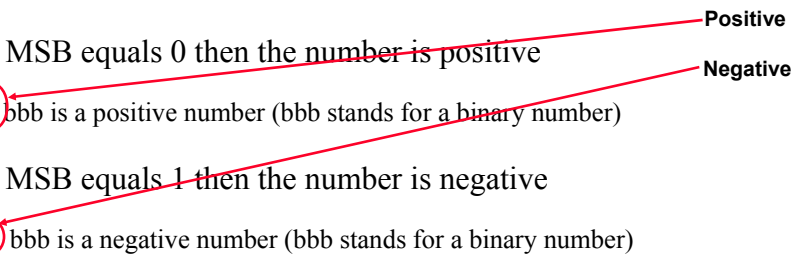

Types of signed representations

- One's complement
- Two's complement

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Binary Subtraction

Requires the negation of a binary number

i.e., A – B becomes A + (-B)

The negation can be performed using the One's complement representation or the Two's complement representation

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Negating Unsigned Binary Using The One's Complement Representation

For positive values there is no difference (no negation is needed)

e.g., positive seven

0111 (unsigned)
0111 (1's complement equivalent)

For negative values negate them by reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0).

e.g., minus six

-0110 (unsigned)
1001 (1's complement equivalent)

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Negating Unsigned Binary Using The Two's Complement Representation

For positive values there is no difference (no negation is needed)

e.g., positive seven

0111 (unsigned)
0111 (2's complement equivalent)

For negative values negate them by reversing (flipping) the bits (i.e., a 0 becomes 1 and 1 becomes 0) *and add one to the result*.

e.g., minus six

-0110 (unsigned)
1010 (2's complement equivalent)

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Interpreting The Pattern Of Bits For Numbers

Bit pattern	Unsigned binary	1's complement	2's complement
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

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Overflow: Unsigned Binary

Occurs when you don't have enough bits to represent a value (wraps –around to zero)

Binary (1 bit)	Value
0	0
1	1

0 0

: :

Binary (2 bits)	Value
00	0
01	1
10	2
11	3

00 0

: :

Binary (3 bits)	Value
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

000 0

: :

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Overflow: Signed

In all cases it occurs do to a "shortage of bits"

Subtraction – subtracting two negative numbers results in a positive number.

e.g. - 7
- 1
+ 7

Addition – adding two positive numbers results in a negative number.

e.g. 7
+ 1
- 8

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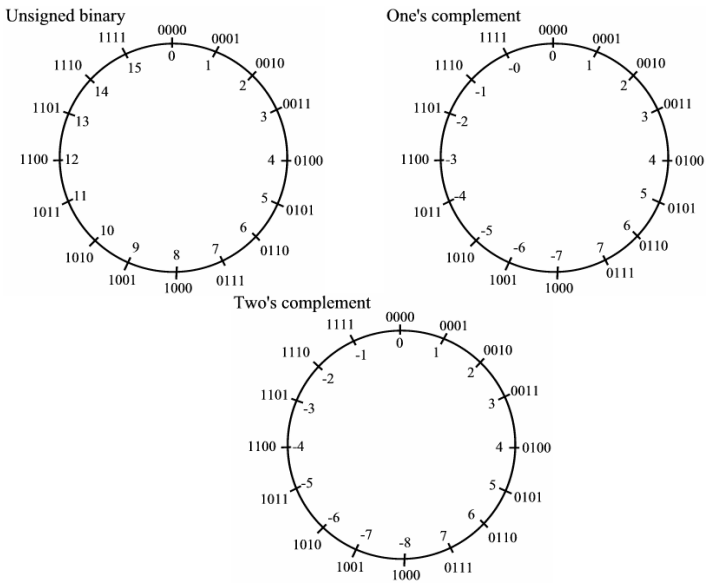
Summary Diagram Of The 3 Binary Representations



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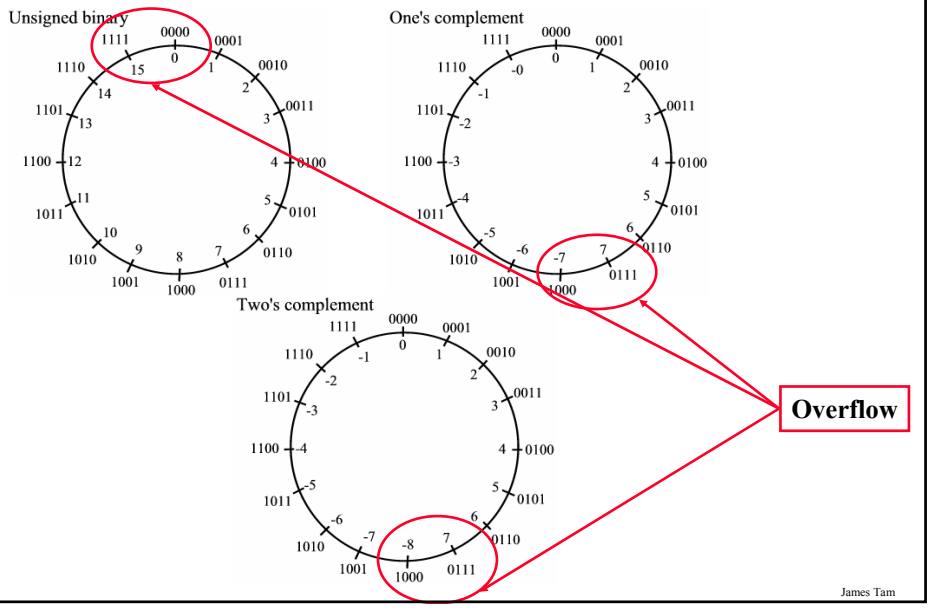

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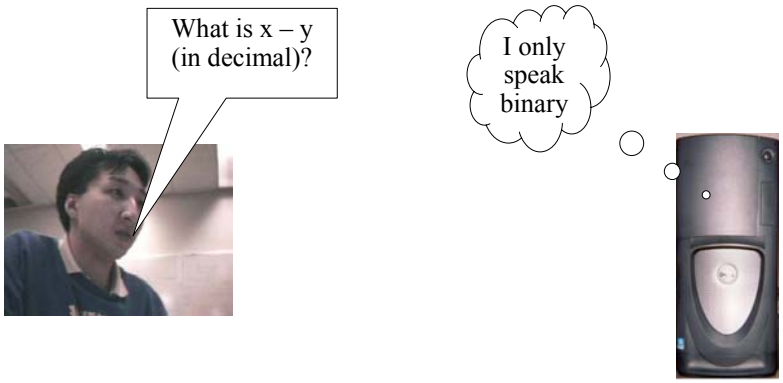
Binary Subtraction Via Negate And Add: A High-Level View



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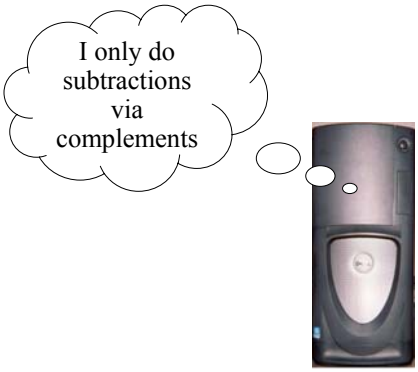
Binary Subtraction Via Negate And Add: A High-Level View



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Binary Subtraction Via Negate And Add: A High-Level View

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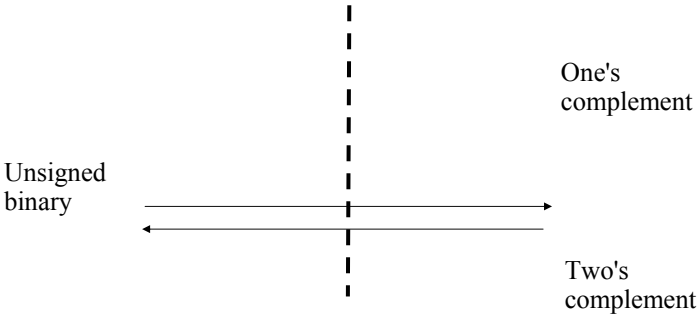
Crossing The Boundary Between Unsigned And Signed



Each time that this boundary is crossed (steps 2 & 7) apply the rule:

1) Positive numbers pass unchanged
2) Negative numbers must be converted (negated)

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Binary Subtraction Through One's Complements

1) *Convert from unsigned binary to a 1's complement representation* (check if it is preceded by a minus sign)
   - If is not preceded by a minus sign, the number is positive (leave it alone).
   - If the number is preceded by a minus sign, the number is negative (negate it by flipping the bits) and remove the minus sign.

2) Add the two binary numbers

3) Check if there is overflow (a bit is carried out) and if so add it back.

4) *Convert the 1's complement value back to unsigned binary* (check the value of the MSB)
   - If the MSB = 0, the number is positive (leave it alone)
   - If the MSB = 1, the number is negative (negate it by flipping the bits) and precede the number with a minus sign

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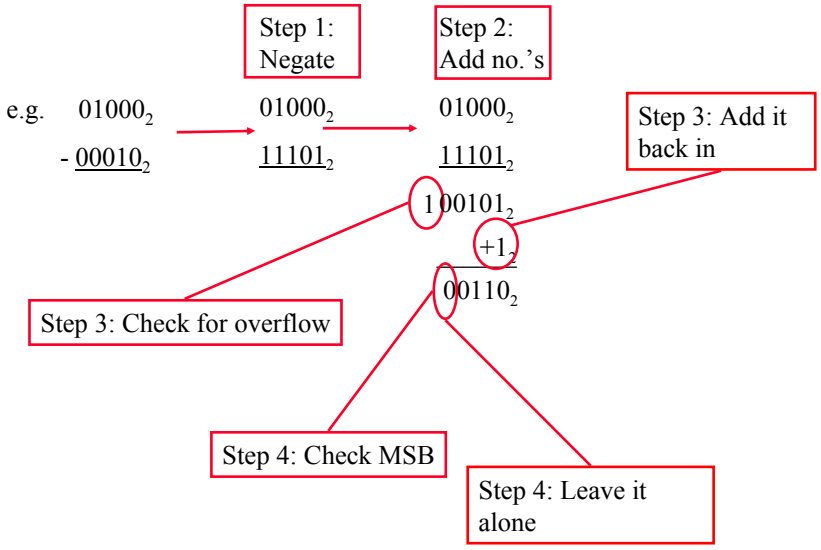
Binary Subtraction Through 1's Complements



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Binary Subtraction Through Two's Complements

1) *Convert from unsigned binary to a 2's complement representation* (check if it is preceded by a minus sign)
   - If is not preceded by a minus sign, the number is positive (leave it alone).
   - If the number is preceded by a minus sign, the number is negative (negate it and discard the minus sign).
     a) Flip the bits.
     b) Add one to the result.

2) Add the two binary numbers

3) Check if there is overflow (a bit is carried out) and if so discard it.

4) *Convert the 2's complement value back to unsigned binary* (check the value of the MSB)
   - If the MSB = 0, the number is positive (leave it alone)
   - If the MSB = 1, the number is negative (negate it and precede the number with a negative sign)
     a) Flip the bits.
     b) Add one to the result.

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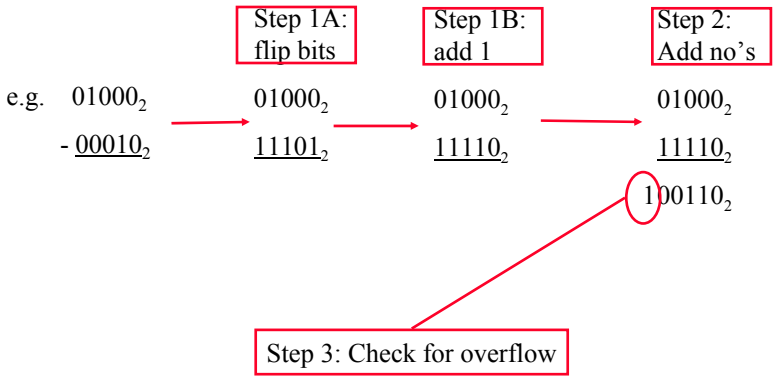
Binary Subtraction Through 2's Complements



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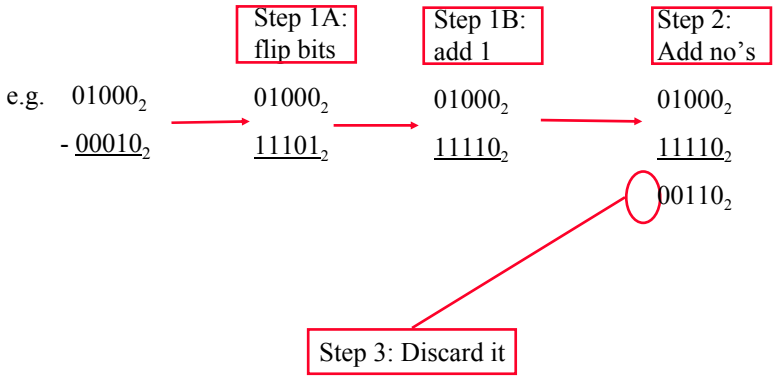

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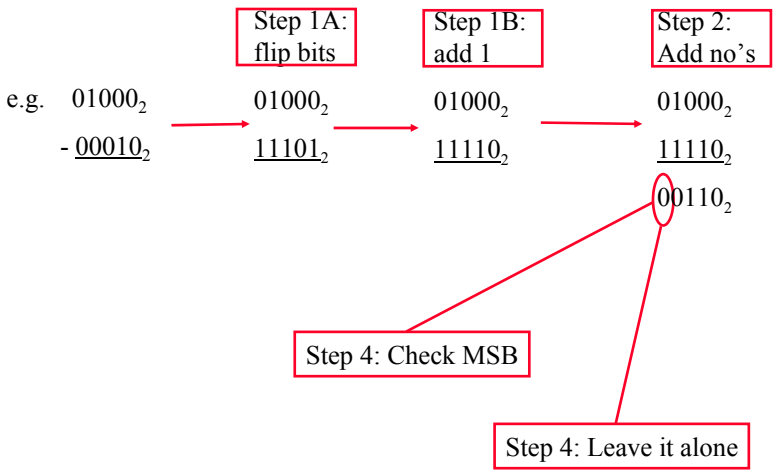
Binary Subtraction Through 2's Complements



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Representing Real Numbers Via Floating Point

Numbers are represented through a sign bit, a mantissa and an exponent

Sign	Mantissa	Exponent

Examples with 5 digits used to represent the mantissa:

- e.g. One: 123.45 is represented as $12345 * 10^{-2}$
- e.g. Two: 0.12 is represented as $12000 * 10^{-5}$
- e.g. Three: 123456 is represented as $12345 * 10^{1}$

Floating point numbers may result in a loss of accuracy!

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You Should Now Know

- How negative numbers are represented using 1's and 2's complements
- How to convert unsigned values to values into their 1's or 2's complement equivalent
- What a signed overflow and why it occurs
- How to perform binary subtractions via the negate and add technique.
- How are real numbers represented through floating point representations

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