Rules of Multi-Channel $\sigma\pi$ Term Logic

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The Term Logic

id:

$$\frac{}{\alpha : A \vdash_{\alpha=\beta} \beta : A}$$

$$\frac{Q \text{ (atomic process)}}{\alpha_1 : A_1 \cdots \alpha_n : A_n \vdash_{\langle \alpha_1, \cdots, \alpha_n \rangle Q \langle \beta_1, \cdots, \beta_m \rangle} \beta_1 : B_1 \cdots \beta_m : B_m}$$

cotupling / tupling:

$$\frac{\{\Gamma, \alpha : A_i, \Gamma' \vdash_{P_i} \Delta\}_{i \in I}}{\Gamma, \alpha : \Sigma A_i, \Gamma' \vdash_{\alpha\{i \mapsto P_i\}} \Delta} \qquad \frac{\{\Gamma \vdash_{Q_i} \Delta, \beta : B_j, \Delta'\}_{j \in J}}{\Gamma \vdash_{\beta\{j \mapsto Q_j\}} \Delta, \beta : \Pi B_j, \Delta'}$$

coprojection / projection:

$$\frac{\Gamma \vdash_Q \Delta, \beta : B_l, \Delta'}{\Gamma \vdash_{\beta[l] \cdot Q} \Delta, \beta : \Sigma B_j, \Delta'} \qquad \frac{\Gamma, \alpha : A_k, \Gamma' \vdash_P \Delta}{\Gamma, \alpha : \Pi A_i, \Gamma' \vdash_{\alpha[k] \cdot P} \Delta}$$

exchange left / right:

$$\frac{\Gamma, \alpha_1 : A_1, \alpha_2 : A_2, \Gamma' \vdash_P \Delta}{\Gamma, \alpha_2 : A_2, \alpha_1 : A_1, \Gamma' \vdash_{P'} \Delta} \qquad \frac{\Gamma \vdash_Q \Delta, \beta_1 : B_1, \beta_2 : B_2, \Delta'}{\Gamma \vdash_{Q'} \Delta, \beta_2 : B_2, \beta_1 : B_1, \Delta'}$$

cut:

$$\frac{\Gamma \vdash_P \Delta, \alpha : X \qquad \beta : X, \Gamma' \vdash_Q \Delta'}{\Gamma, \Gamma' \vdash_{P \;_{\alpha;\beta} Q} \Delta, \Delta'}$$

Cut Elimination

$$P \;{}_{\alpha};{}_{\beta}\; \gamma\{i \mapsto Q_i\} \quad \overset{\gamma \neq \beta}{\Longrightarrow} \quad \gamma\{i \mapsto P \;{}_{\alpha};{}_{\beta}\; Q_i\}$$

$$\gamma\{i \mapsto Q_i\} \;{}_{\alpha};{}_{\beta}\; P \quad \overset{\gamma \neq \alpha}{\Longrightarrow} \quad \gamma\{i \mapsto Q_i \;{}_{\alpha};{}_{\beta}\; P\}$$

$$P \;{}_{\alpha};{}_{\beta}\; \gamma[i] \cdot Q \quad \overset{\gamma \neq \beta}{\Longrightarrow} \quad \gamma[i] \cdot (P \;{}_{\alpha};{}_{\beta}\; Q)$$

$$\gamma[i] \cdot Q \;{}_{\alpha};{}_{\beta}\; P \quad \overset{\gamma \neq \alpha}{\Longrightarrow} \quad \gamma[i] \cdot (Q \;{}_{\alpha};{}_{\beta}\; P)$$

$$\alpha[i] \cdot P \;{}_{\alpha};{}_{\beta}\; \beta\{i \mapsto Q_i\} \quad \Longrightarrow \quad P \;{}_{\alpha};{}_{\beta}\; Q$$

$$\alpha\{i \mapsto Q_i\} \;{}_{\alpha};{}_{\beta}\; \beta[i] \cdot P \quad \Longrightarrow \quad Q_i \;{}_{\alpha};{}_{\beta}\; P$$

$$\alpha = \beta \;{}_{\beta};{}_{\gamma}\; \langle \cdots \gamma \cdots \rangle P \langle \cdots \rangle \quad \Longrightarrow \quad \langle \cdots \alpha \cdots \rangle P \langle \cdots \rangle$$

$$\alpha = \beta \;{}_{\beta};{}_{\gamma}\; \gamma = \delta \quad \Longrightarrow \quad \alpha = \delta$$

Identities

$$\alpha[i] \cdot \beta[j] \cdot P \quad \overset{\alpha \neq \beta}{\Longleftrightarrow} \quad \beta[j] \cdot \alpha[i] \cdot P$$

$$\alpha[i] \cdot \beta\{j \mapsto P_j\} \quad \overset{\alpha \neq \beta}{\Longleftrightarrow} \quad \beta\{j \mapsto \alpha[i] \cdot P_j\}$$

$$\alpha\{i \mapsto \beta\{j \mapsto P_{ij}\}\} \quad \overset{\alpha \neq \beta}{\Longleftrightarrow} \quad \beta\{j \mapsto \alpha\{i \mapsto P_{ij}\}\}$$