Course Proposal: Homotopy Type Theory
Winter 2016

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General

In winter 2015, "Homotopy Type Theory" was offered with great success to investigate the recent description of Martin-Löf type theory as a syntax for abstract homotopy theory. The proposal is to repeat the course with some refinement and further developments. The the course covers abstract homotopy theory, its relation to type theory, and advanced topics and open problems in homotopy type theory with a focus on categorical homotopy theory.

Students coming out of the course:

- Will have an understanding of abstract homotopy theory;
- Will have an understanding of dependent type theory and models of dependent type theory;
- Will understand the interpretation of equality and equivalence in homotopy type theory and the axiom of univalence;
- Be in a position to be able to apply their knowledge to advance the field.

Outline

The course will cover the following topics (in order):

- Ends and CoEnds (suggested reference: MacLane "Categories for the Working Mathematician")
- Kan extensions (suggested references: Marina Lehner's notes[1]; MacLane "Categories for the Working Mathematician"; Awodey "Category Theory" (for the characterization of $[\mathbb{X}^{\mathsf{op}}, \underline{\mathsf{Set}}]$ as the colimit completion of $\mathbb{X}$))
- Introduction to dependent type theory and categorical models (suggested references: "Homotopy Type Theory" Institute for Advanced Study; Robert Seely's paper[2]; Jacobs "Categorical Logic and Type Theory")
- Introduction to factorization systems, model structure, and ∞ categories (suggested references: Riehl "Categorical Homotopy Theory"; Lurie "Higher Topos Theory"; Hirschhorn "Model Categories and Their Localizations")

[1] www.math.harvard.edu/theses/senior/lehner/lehner.pdf
[2] http://www.math.mcgill.ca/rags/LCCC/LCCC.pdf

- Model structures on simplicial sets, topological spaces, and chain complexes, and their Quillen equivalence (suggested references: Goerss and Jardine "Simplicial Homotopy Theory")
- Models of homotopy type theory (suggested references: Garner's model[3][4][5]; Warren's model[6])
- Univalence and modelling univalence (suggested reference: Warren[7])

Course Structure and Grading Scheme

After some introductory lectures, the course will be structured around student lectures and some guest lectures by Jonathan Gallagher and Chad Nester. Each student will be responsible for presenting lectures on a segment of the above material using the suggested references as background material. Over the course of the semester each student will be responsible for up to 5 ninety minute lectures of material. In addition, each student will be expected to develop detailed notes on two topics covered by the course for evaluation. The course credit will be allocated as follows:

Presentations: The presentations will be worth 50% of the course. Each presentation will be assigned a mark out of 10 and the two best presentations will be used for the actual grade.

Written notes: Each of the two sets of detailed notes, which must be submitted, will be graded out of 25% making up the balance of the marks for the course.

Prerequisites

This is a theoretical course and a basic level of mathematical maturity is required. Specifically the following are required:

1. A course on category theory (e.g. cpsc617)
2. A basic understanding of type theory and proof theory (e.g. cpsc521 or phil473)

[3] http://comp.mq.edu.au/~rgarner/Papers/Uppsala.pdf
[4] http://comp.mq.edu.au/~rgarner/Papers/Idtype.pdf
[5] http://comp.mq.edu.au/~rgarner/Papers/Topsimp.pdf
[6] http://mawarren.net/papers/phd.pdf
[7] http://arxiv.org/pdf/1210.5658v1.pdf