THE UNIVERSITY OF CALGARY

FACULTY OF SCIENCE

FINAL EXAMINATION

COMPUTER SCIENCE 417

December, 2007 Time: 2 hrs.

Instructions

The exam contains questions totalling 100 points. Answer all questions. This exam is closed book.

25 marks

1. Given the following datatype for terms:

```
data Term = Var Int
          | Node String [Term]
```

   and following datatype for exceptions:

```
data SF a = FF | SS a
```

   (a) Explain what a *unifier* and what a *most general unifier* of two terms is.

   (b) Create an instance of a monad, the exception monad, for the `SF` type.

   (c) Write a Haskell function to find the most general unifier of two terms.

25 marks

2. In the λ-calculus:

   (a) What is a fixed point combinator? Give an example of a fixed point combinator and show that it has the desired property.

   (b) Explain how fixed point combinators are used to encode general recursion.

   (c) Explain how one may represent binary trees

   ```
   data BTree a = Leaf a
                | Node (BTree a) (BTree a)
   ```

   in the λ-calculus.

   (d) Assuming that one has a representation of numbers and of their basic functions (such as addition) describe how to encode in the λ-calculus the function to sum the leaves of the tree.

   Do you need to use general recursion?

25 marks

3. Given an algebraic system with a binary operation $\bullet$ and constants b, c, and k satisfying:

$$\begin{aligned} ((\mathsf{b} \bullet x) \bullet y) \bullet z &\Rightarrow x \bullet (y \bullet z) \\ ((\mathsf{c} \bullet x) \bullet y) \bullet z &\Rightarrow (x \bullet z) \bullet y \\ (\mathsf{k} \bullet x) \bullet y &\Rightarrow x \end{aligned}$$

   (a) Prove that this rewriting system is terminating.

   (b) Explain what it means for a rewriting system to be confluent and prove that this system is confluent.

$$\frac{}{x : P, \Gamma \vdash x : P}\ \text{proj}$$

$$\frac{x : P, \Gamma \vdash t : Q}{\Gamma \vdash \lambda x.t : P \to Q}\ \text{abst}$$

$$\frac{\Gamma \vdash f : P \to Q \quad \Gamma \vdash t : P}{\Gamma \vdash (ft) : Q}\ \text{app}$$

Table 1: Type judgements

$$\frac{}{x : P, \Gamma \vdash x : Q}\ \rhd P = Q$$

$$\frac{x : P, \Gamma \vdash t : R}{\Gamma \vdash \lambda x.t : Q}\ P, R \rhd Q = P \to R$$

$$\frac{\Gamma \vdash f : R \quad \Gamma \vdash t : P}{\Gamma \vdash (ft) : Q}\ P, R \rhd R = P \to Q$$

Table 2: Type judegements with type equations

25 marks

4. Using the judgements for type inference in table 2:
   (a) Show that the term, $\lambda xy.(yx)yx$, cannot be typed in the simply typed lambda calculus.
   (b) Show that $\mathsf{c} = \lambda xyz.xzy$ can be typed in the simply typed lambda calculus and provide its most general type.