Supplementary Material for: "Euclidean Rhythms Found in English Poetry and Lyrics"

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Abstract

We provide additional details and examples to support and clarify the study presented in the main paper.

The Bjorklund Algorithm

The Bjorklund algorithm was originally developed to address an engineering problem in nuclear physics systems, specifically in the timing control of components within Spallation Neutron Source (SNS) accelerators. More generally, the algorithm can be applied to distribute a fixed number of events as evenly as possible across a set of time slots [3]. In simpler terms, consider k total time intervals in which n pulses are to be placed, each interval either contains a pulse (denoted by 1) or does not (denoted by 0). The objective is to arrange these 1s and 0s such that the pulses are distributed as evenly as possible.

Toussaint [3] demonstrated that the Bjorklund algorithm can be used to generate Euclidean rhythms, noting that the Euclidean algorithm shares the same structural properties as the Bjorklund algorithm. The full procedure used in the main paper to generate a Euclidean rhythm $E(n, k)$ is presented in Algorithm 1.

Algorithm 1 The Bjorklund Algorithm

```
function BJORKLUND(group1, group2)
    if LEN(group2) = 1 then
        return group1 + group2
    end if
    result ← empty list
    while LEN(group1) and LEN(group2) do
        result.append(group1.pop() + group2.pop())
    end while
    return BJORKLUND(result, group1 + group2)
end function
function E(n, k)
    group1 ← [1]^n
    group2 ← [0]^(k−n)
    return BJORKLUND(group1, group2)
end function
```

Examples of Common English Metrical Feet

The following provide examples of the five most common metrical feet in English poetry, demonstrating how they are represented in terms of stressed and unstressed syllables, as well as their corresponding beat patterns. Each table includes a line from a well-known poem that exemplifies the specific metrical foot being illustrated.

shall I	comPARE	thee TO	a SU	mmer's DAY
˘ ˊ	˘ ˊ	˘ ˊ	˘ ˊ	˘ ˊ
da DUM	da DUM	da DUM	da DUM	da DUM

Table 1: *The iamb appears in the opening of Shakespeare's Sonnet 18*

BY the	SHORES of	GITche	GUmee
ˊ ˘	ˊ ˘	ˊ ˘	ˊ ˘
DUM da	DUM da	DUM da	DUM da

Table 2: *The trochee appears in the opening of Longfellow's The Song of Hiawatha*

the asSYR	ian came DOWN	like the WOLF	on the FOLD
˘ ˘ ˊ	˘ ˘ ˊ	˘ ˘ ˊ	˘ ˘ ˊ
da da DUM	da da DUM	da da DUM	da da DUM

Table 3: *The anapest appears in Byron's The Destruction of Sennacherib*

HALF a league	HALF a league
ˊ ˘ ˘	ˊ ˘ ˘
DUM da da	DUM da da

Table 4: *The dactyl appears in the opening lines of The Charge of the Light Brigade by Alfred Tennyson*

all REAdy	to PUT up	the TENTS for	my CIRcus.
˘ ˊ ˘	˘ ˊ ˘	˘ ˊ ˘	˘ ˊ ˘
da DUM da	da DUM da	da DUM da	da DUM da

Table 5: *The amphibrach appears in If I Ran the Circus by Dr. Seuss*

Geometric Representation of Musical Rhythms

Toussaint [2] presented a geometric representation of cyclic musical rhythms by setting the music pulses clockwise on a circle. Each pulse, whether denoting a note or a rest, corresponds to a vertex on the circle starting from an initial note. By connecting the note vertices, a convex polygon is formed. Figure 1a demonstrates this geometric representation for the periodic pattern 10101010 on the circle where 1 represents a note and 0 represents a rest. The starting note is marked in blue, and the rhythm plays clockwise. In the figure, the edges are assigned weights that correspond to the geodesic distance between consecutive notes. The distance between two notes i and j is $d(i, j) = |V(i, j)| + 1$. where $V(i, j)$ is the set of all vertices between i and j on the circle. It is important to note that the geodesic distance represents the shortest path between any two notes, as an alternative shorter path may be available when traversing in the opposite direction (i.e., counterclockwise) around the circle. Consequently, the distance cannot exceed $\lfloor \frac{N}{2} \rfloor$, where N denotes the total number of vertices. The clockwise sequence of adjacent distances between notes is called the inter-onset

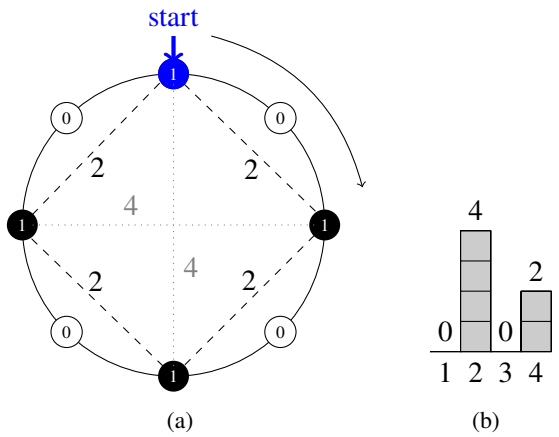

Figure 1: *Geometric representation of* 10101010 *(a) on a circle and (b) its interval histogram.*

interval (IOI) pattern and it is in this case $I = 2222$. If all the distances between notes are included, it is often referred to as the *full* interval vector and often represented visually with a histogram [4] as shown in Figure 1b. The interval histogram shows the number of occurrences for each possible distance between notes.

Within Stress Patterns

When reading Shakespeare's *Sonnet 18*, try to align the syllables with the two-step rhythmic period. Whether you imagine walking at a regular pace or with a skipping stride, the natural flow of the iambic rhythm remains perceptible. Ultimately, the difference between the sounds of stressed and unstressed syllables is not definitively translated in our minds to a difference in duration, but perhaps intensity (loudness) and pitch (shrillness) play a role [1]. This raises an important question: are these abstractions inherently natural?

To illustrate this issue, consider the following five-syllable snippets from *Sonnet 18*: "to a summer's day" and "fair some time decline." Both phrases contain five syllables with the same stress pattern (´ ˘ ´ ˘ ´), which can be traditionally abstracted by the mnemonic DUM da DUM da DUM.

to	a	su	mmer's	day
fair	some	time	dec	line
DUM	da	DUM	da	DUM

If we attempt to align the beats strictly with the syllables as abstracted above, we might be forced to unnaturally drawl the syllables "to" and "su" in "to a summer's day" and to slur "some" in "fair some time decline." Such adjustments disrupt the natural flow of language and could cause a singer to stumble over the words if the verses were meant to be sung with the percussion. The singer might then feel compelled to slow down or rush certain syllables in order to match the beat.

Solving this issue is particularly useful for song lyrics, where we believe that considering the natural rhythm of language is essential. We propose that a more refined approach is needed, one that adjusts the positions of the beats at a more detailed, sub-syllabic level.

To further clarify, consider aligning the same phrases above with a specific beat pattern. Although both phrases have the same number of syllables and share an identical stress pattern, the question arises: Is this similarity sufficient for them to align with an identical beat pattern? Suppose we adopt a beat pattern consisting of three consecutive beat pulses followed by two beat-rest, represented as 1111010 or *da da da dum dum*. When aligning the phrases with this pattern, we obtain:

to	a	su	mmer's	day
fiar	some	time	dec	line
da	da	da	DUM	DUM

You may notice that the first phrase naturally conforms to the beat pattern, whereas the second phrase would require an unnatural slurring of its first three syllables. This discrepancy suggests that aligning syllables with beats based solely on stress patterns may not be sufficient.

Moving beyond syllable-level emphasis, we now turn our attention to the patterns formed by the sequence of consonants and vowels within syllables. A syllable in English consists of three components: an onset, a nucleus, and a coda (see Figure 2). A syllable that lacks a coda is known as *open syllable* such as the word *me*, whereas one without an onset is referred to as a *null onset* syllable such as the word *it* (see Figure 3).

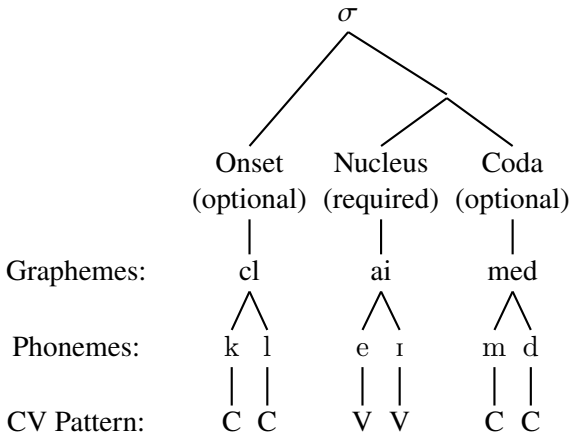

Figure 2: *Syllable structure in English, using the one-syllable word "claimed" as an example.*

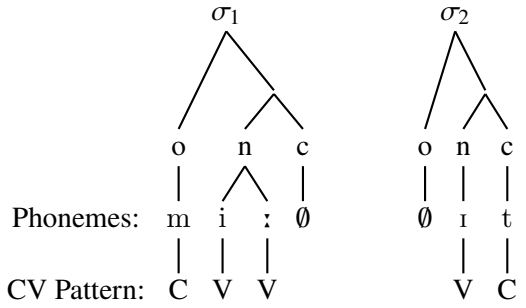

Figure 3: *Examples of syllable structures: "me" (σ_1) as an open syllable and "it" (σ_2) as a syllable with a null onset.*

Grapheme-to-Beat Mapping

The following is a detailed example of how to convert a line of text from its graphemes to beat patterns to circles, using the line "Yesterday comes suddenly" from the song "Yesterday" by the *Beatles* as an example. The line consists of seven syllables, each represented by a node σ_i in the tree structure shown in Figure 4.

Each syllable is further broken down into its onset, nucleus, and coda components. The phonetic transcription of the line is provided in the International Phonetic Alphabet (IPA). Once text is converted into phonemes, the next step is to represent these phonemes as a consonant–vowel sequence.

In English, the distinction between consonants and vowels from phonemes is generally straightforward, yet it can be blurry in certain cases. For instance, an r-colored vowel may behave like a vowel extension when it occurs at the end of a word (e.g., in the word "hammer") or as the beginning of a vowel sound in the next syllable (e.g., in "hammer it"). In the latter case, the combination of the "r" and the following vowel forms as a vowel onset, which corresponds to a note in the rhythm.

Following this, the CV sequence is converted into beat patterns. Algorithm 2 illustrates the conversion from the CV sequence to the beat pattern. For the sake of simplicity, the algorithm assumes that only short vowels are marked as 'V'. In contrast, long vowels, diphthongs, and r-colored vowels are marked as 'VC'. This is because in our scheme, the second component in long vowels, diphthongs, and r-colored vowels functions similarly to a consonant by being represented as a rest in the beat pattern, except when it is immediately followed by another vowel. In such instances, it is treated as a vowel onset that carries a beat note. All other consonants are simply denoted as 'C' in the consonant–vowel sequence.

Consequently, a "1" in the generated beat pattern indicates a note, specifically at a vowel onset (CV), while a "0" denotes a rest. Moreover, if a vowel appears at the beginning of a sentence, it is assumed that a glottal stop precedes it. This glottal stop is treated as a vowel onset, and the vowel is assigned a beat note accordingly.

Algorithm 2 CV Pattern Conversion

Require: String s consisting of characters 'C' and 'V'
Ensure: Binary string representing the converted pattern

```
Initialize empty string output
i ← 0
while i < length of s do
    if s[0] is a 'V' then                          ▷ Assume the first vowel is preceded by a glottal stop
        Append '1' to output
    else if s[i] = 'C' AND s[i + 1] = 'V' then     ▷ i + 1 ≤ length of s
        Append '1' to output
        i ← i + 1                                  ▷ Skip the next character
    else
        Append '0' to output
    end if
    i ← i + 1
end while
return output
```

The resulting beat pattern of the line "Yesterday comes suddenly" is 101101011010 (see Figure 4). This pattern is a Euclidean rhythm and can be generated by $E(7, 12)$ (see Algorithm 1). Figure 5a illustrates the geometric representation of the beat pattern discussed in the main paper. The starting note is marked in blue, and the rhythm plays clockwise. Figure 5b shows the interval histogram of the beat pattern, illustrating unique non-zero multiplicities of all possible distances, meaning that the rhythm is Winograd and Erdős deep.

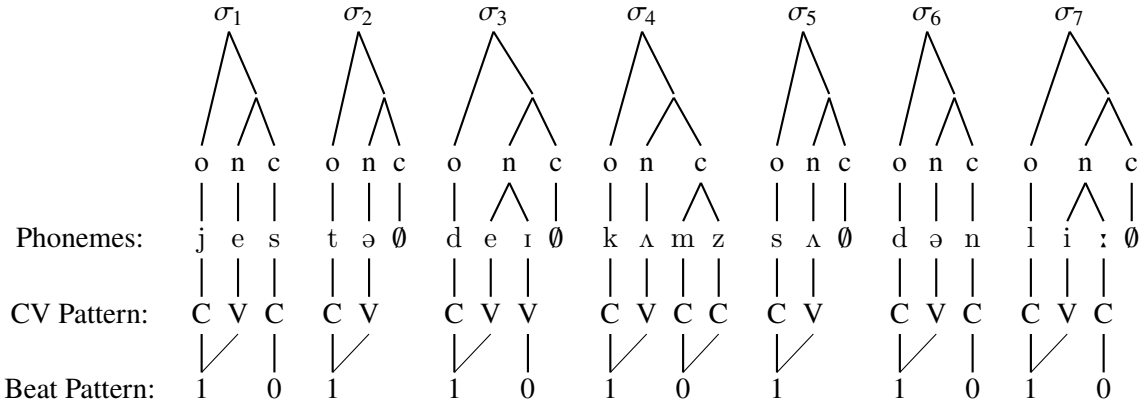

Figure 4: *The process of converting the second refrain "Yesterday comes suddenly".*

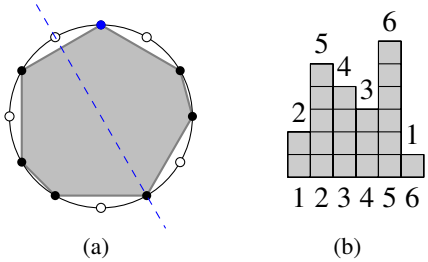

Figure 5: *(a) The convex polygon of* 101101011010 *which represents the beat pattern for "Yesterday comes suddenly" and (b) its interval histogram.*

Examples of Euclidean and Random Rephrasing Used in our Human Subject Study

Original	**Euclidean**	**Random**
Live long and lovely but yet grow no less *–Robert Herrick*	Wishing you a long and beautiful life without decrease	May you live long and full of grace, without diminishing
Gone ! As a pirate flies before the wind *–Oliver Wendell Holmes*	Away ! As a pirate whisked away by the howling wind	Disappeared ! Much like a pirate swiftly carried by the wind
I cannot write of turmoil, I cannot write of strife *–Amy Redpath Roddick*	The unrest evades my pen, the discord eludes my lines	I am unable to pen about chaos, I am unable to capture conflict
Heard when all the world was just a crazy flight of dreams *–John Drinkwater*	Recalled when the whole world was a fantastical dreamland	Perceived when the universe was a chaotic dream sequence

Table 6: *Examples of original lines from real poets used in our human subject study in the main paper, along with their corresponding Euclidean and random rephrases.*

References

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[2] G. Toussaint. “A Mathematical Analysis of African, Brazilian, and Cuban Clave Rhythms.” *Bridges Conference Proceedings,* Towson, Maryland, Jul. 27–29, 2002, pp. 157–168. https://archive.bridgesmathart.org/2002/bridges2002-157.html.

[3] G. Toussaint. “The Euclidean Algorithm Generates Traditional Musical Rhythms.” *Bridges Conference Proceedings,* Banff, Alberta, Jul. 31 to Aug. 3, 2005, pp. 47–56. https://archive.bridgesmathart.org/2005/bridges2005-47.html.

[4] G. T. Toussaint. *The Geometry of Musical Rhythm: What Makes a “Good” Rhythm Good?* CRC Press, 2019.