On the Impact of Dynamic Jamming on End-to-End Delay in Linear Wireless Networks

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***Abstract*—The effectiveness and straightforward implementation of jammers make them an essential security threat for wireless networks. In particular, in order to increase the battery lifetime of the jammers and add spatial and temporal randomness to the jamming signal, it is common for the adversaries to use duty cycling strategies. In this case, the state of each wireless link is governed by the dynamics of multiple stochastic jammers affecting each link. Since it is possible that each jammer can interrupt communication over multiple links, the up-down dynamics of different links can be spatially and/or temporally correlated. Dynamic networks have been considered widely in the literature; however, most analyses have ignored these challenging dependencies. In this paper, we consider communication over a linear network in the presence of duty cycling jammers. We model the process associated to each link in both slotted time and continuous-time regimes, and evaluate the exact end-to-end latency of the network in each case.**

I. INTRODUCTION

Wireless networks, due to their broadcast nature, are susceptible to many security attacks. Among them, passive eavesdropping attacks have attracted a lot of attention in the literature (e.g. see [1] and references therein). Nevertheless, active jamming attacks are a key component of any security scheme, as jamming can severely disrupt the network performance, and thus are of interest in this work. In particular, jamming the physical layer is one of the simplest and most effective attacks, as any cheap radio device can broadcast electromagnetic radiation to block the communication channel [2]. They may not only alter the interference level of the channel as commonly assumed in modeling of the malicious jammers, but also can greatly impact receiver operation by compressing the dynamic range of the receiver's front-end [3].

Now consider a multi-hop wireless network such as a mesh network. In order to maximize the area under the jamming attack, instead of using a few jammers, an adversary can spread many jammers at arbitrary and random locations in the network. In this case, the jammers rely on battery power, which is limited, and hence it is essential for the jammers to seek methods to reduce their energy consumption and increase their jamming lifetime. A simple way to increase the battery lifetime of the jammers, while keeping their design simple, is applying random duty cycling technique [4]. Duty cycling strategies, where each node of the network switches frequently between an active mode and a sleep mode, have been extensively used as a means of energy conservation in wireless networks [5]. An important performance metric in networks with duty cycling nodes is the end-to-end latency for transmitting messages. Because of duty cycling, some nodes along the transmission path of a message may be in sleep mode and hence the message has to wait for those node to become active. Network latency with duty cycling nodes, in both exact and asymptotic regimes has been studied in the literature [6], [7]; however, wireless communication in the presence of duty cycling jammers and the impact of them on the network latency is not investigated.

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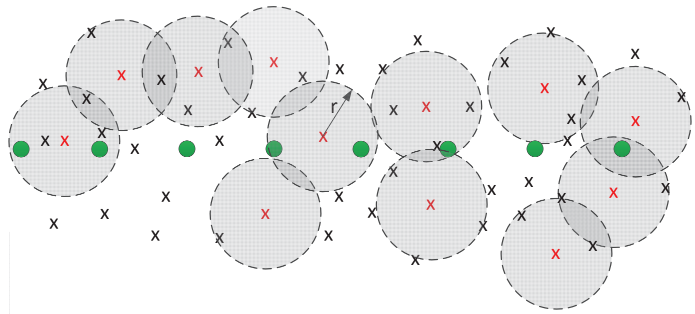

Fig. 1. The linear network in the presence of on-off jammers. The network nodes are shown by green circles, the silent jammers by black crosses, and the active jammers by red crosses. The jammers switch between active and silent modes frequently. Each active jammer can wipe off an area of radius r around it, and thus any node in this area cannot receive the message.

This paper considers multi-hop wireless communication in a linear network in the presence of such random duty cycling jammers (Figure 1). Using duty cycling jammers over a network would enable the adversary to interfere with the communication of many nodes over a large area with low power and complexity. We assume that each jammer, when it is active, affects an area with a fixed radius around it such that the receivers located in that area cannot receive any message. Because of the stochastic nature of the jammers, and the fact that each jammer can possibly affect multiple nodes, the the processes that describe the up and down links across the network are potentially temporally and/or spatially correlated. As a result, the network behaves like a dynamic graph with stochastic links. While dynamic networks, with variety of applications in different fields, have widely been studied in the literature [8], [9], only a few studies have considered such dependencies [10].

In this work, the exact end-to-end latency of the transmission of a message for two cases of synchronous and asynchronous jammers is characterized. In the first part of the paper, we assume that the time is slotted, and in every time slot, each jammer, which can affect multiple nodes and thus cause correlation between the dynamics of different links, is active with probability p and silent with probability $1-p$. In the second part of the paper, we assume that each jammer changes its state asynchronously and at random times, such that the duration of an active period and a silent period are two independent exponential random variables with rates λ and μ, respectively. Therefore, unlike the first part, in the second part we model the dynamics of each link by a continuous time renewal process and find the latency of the network. The theoretical framework developed in this work can be applied to other systems such as interference channels and duty cycling networks.

The rest of the paper is organized as follows. Section II describes the system model. The latency of synchronous and asynchronous on-off jammers is studied and characterized in Sections III and IV, respectively. In Section V, conclusions and ideas for future work are discussed. The proofs of the theorems and lemmas are presented in Section VI.

II. System Model

Suppose our system consists of $n+1$ nodes $V = \{v_0, v_1, v_2, ..., v_n\}$ located on a linear network. Without loss of generality we assume that the consecutive nodes are located at unit distance from each other. The node v_0 transmits a message to the node v_n hop by hop via the intermediate nodes. Some jammers are present over the surface that contains the linear network at arbitrary locations due to a homogeneous Poisson point process with density σ. Each jammer decides independently of other jammers to send a jamming signal or be silent. Suppose when a jammer is active, it can wipe out a region of radius r around itself, i.e. no message can be received in that region. Hence, a link is "open" when there is no active jammer in the region of radius r around the receiver node, and it is "close" otherwise. We assume that when the message arrives at a node and finds the next link open, the waiting time at this node is zero. We assume that the propagation delay and the queuing delay are zero. Hence, in the absence of jammers the end-to-end latency of the network, i.e. the time that a message can travel from the source to the destination, is zero. Our goal is to find the latency of communication between v_0 and v_n in the presence of the duty cycling jammers.

Two cases of synchronous and asynchronous jammers are considered. In the first part, we assume that the time is divided into time slots. In every time slot, each jammer is in ON state (sends the jamming signal) with probability p and is in OFF state (silent) with probability $1-p$. At the beginning of each time slot each jammer chooses its state independently of its previous state and of the state of the other jammers. In the second part of the paper, we assume that each jammer changes its state asynchronously and at random times. The duration of time that each jammer is in ON state is an exponential random variable with rate λ and the duration of time that each jammer is in an OFF state is an independent exponential random variable with rate μ. The end-to-end latency of the hop-by-hop transmission of a message from the source (v_0) to the destination (v_n) for each case is investigated in the following sections.

III. Synchronous On-Off Jammers

A. Synchronous On-Off Jammers with Small Jamming Range

In this section we assume that the jammers change their state simultaneously and independently at the beginning of each time slot such that each jammer is on with probability p and is off with probability $1-p$. In order to gain insight into problem, we first assume that the jamming range of each jammer is small such that the jammers affecting one node are independent from the jammers affecting the other nodes, i.e. $r < 1/2$. This case corresponds to the case of jammers operating in a low power regime or in an area with high signal attenuation. The average latency of sending a message from v_0 to v_n is presented in the following lemma.

Lemma 1. The latency of a linear n-hop network in the presence of on-off jammers distributed according to a Poisson point process with density σ and the jamming radius $r < 1/2$ is,

$$E[w] = n(e^{\frac{\sigma p \pi r^2}{1-p}} - 1).$$

Proof: The proof is presented in Appendix A. ■

B. Synchronous On-Off Jammers with Arbitrary Jamming Range

Similar to the previous section, here we assume that the jammers are synchronous. However, the transmission range of each jammer can be large and thus each jammer can affect multiple nodes. Suppose that the process associated with the i^{th} link is denoted by $l_i = \{l_i(k)\}_{k=-\infty}^{k=\infty}$, where k is discrete time. $l_i(k) = 0$ when at time slot k the link between v_{i-1} and v_i is open (all the jammers in the region of radius r around v_i are off) and $l_i(k) = 1$ otherwise. Since the jamming range can be large ($r > 1/2$), some jammers can affect multiple nodes and thus some dependency between the processes of different links exists.

In this case, the following lemma shows the end-to-end latency of the network.

Theorem 1. Latency of a linear n-hop network in the presence of synchronous independent on-off jammers with density σ is:

$$E[w] = e^{\frac{\sigma p \pi r^2}{1-p}} - 1 + (n-1)(e^{\frac{\sigma p \pi r^2}{1-p}} - e^{\frac{\sigma p A_C}{1-p}})$$

where p is the probability of a jammer being on, r is the jamming radius of each jammer, and,

$$A_C = 2r^2 \cos^{-1}(\frac{1}{\sqrt{2}r}) - \sqrt{2r^2 - 1}$$

Proof: The proof is presented in Appendix B. ■

IV. Asynchronous On-Off Jammers

In this section, unlike the previous sections that all the jammers were changing their states simultaneously, each jammer switches between ON state (when it transmits the jamming signal) and OFF state (when it is silent) independently from the other nodes. Also, the duration of an ON state and an OFF state for each jammer is no longer fixed, and are two independent exponential random variables with rates μ and λ, respectively. This model, in particular, describes the simple jammers that are spread over an area without any connection to each other and without any synchronization between them, and switch between ON and OFF modes to save energy and increase their jamming lifetime. In this case, we show the random process of the i^{th} link by $l_i = \{l_i(t)\}_{t=-\infty}^{t=\infty}$, where t is continuous time. When the link between v_{i-1} and v_i is open (all the jammers in the region of radius r around v_i are off) at time t, $l_i(t) = 0$ and $l_i(i) \neq 0$ otherwise. The reason that we consider $l_i(t) \neq 0$ instead of $l_i(t) = 1$ will be discussed in the proof of Theorem 2. Theorem 2 characterizes the average end-to-end latency of a linear network in the presence of such jammers. For simplicity we assume that $r < 1/2$, i.e. each jammer affects at most one node. Furthermore, although in general μ and λ can be different, to obtain a closed form formula we assume $\lambda = \mu$.

Theorem 2. The average end-to-end latency of a linear network in the presence of asynchronous on-off jammers is,

$$= \frac{ne^{-\sigma\pi r^2}}{2\lambda}(\text{Ei}(\sigma\pi r^2/2) - \ln(\sigma\pi r^2/2)).$$

Proof: Proof is presented in Appendix C. ■

V. Conclusion and Future work

In this paper, multi-hop communication over a linear wireless network in the presence of synchronous and asynchronous duty cycling jammers is considered. In the case of synchronous jammers, the dynamics of each link is modeled as a discrete time process, and the exact expected end-to-end message delay is evaluated. In the case of asynchronous jammers, the dynamics of each link is modeled by a continuous time birth-and-death process, and the exact expected latency of sending a message due to presence of jammers is characterized. Modeling the link dynamics of a 2D grid network in the presence of stochastic jammers and computing the end-to-end latency are currently under investigation.

VI. Appendices

Appendix A

Suppose that the area of the jamming region around v_1 is A. Let us denote the number of jammers in this region by M. Since the jammers are distributed according to a Poisson point process,

$$P(M = m) = \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

The proof of Lemma 1 follows.

Proof: The waiting time at v_0 given that $M = m$ jammer exist in the region around v_1 follows a geometric distribution,

$$p(w_0 = k|M = m) = (1 - (1-p)^m)^k(1-p)^m.$$

Thus, using the law of total probability,

$$\begin{aligned} p(w_0 = k) &= \sum_{m=1}^{\infty} p(w_0 = k|M = m)P(M = m) \\ &= \sum_{m=1}^{\infty} (1 - (1-p)^m)^k(1-p)^m \frac{e^{-\sigma A}(\sigma A)^m}{m!} \end{aligned}$$

Hence, the expected value of the waiting time at v_0 is,

$$\begin{aligned} E[w_0] &= \sum_{k=0}^{\infty} kp(w_0 = k) \\ &= \sum_{k=0}^{\infty} k \sum_{m=1}^{\infty} (1 - (1-p)^m)^k(1-p)^m \frac{e^{-\sigma A}(\sigma A)^m}{m!} \\ &= \sum_{m=1}^{\infty} \frac{e^{-\sigma A}(\sigma A)^m}{m!} \sum_{k=0}^{\infty} k(1 - (1-p)^m)^k(1-p)^m \\ &= \sum_{m=1}^{\infty} \left(\frac{1}{(1-p)^m} - 1\right) \frac{e^{-\sigma A}(\sigma A)^m}{m!} \\ &= \frac{e^{-\sigma A}}{e^{-\sigma A/(1-p)}}(1 - e^{-\sigma A/(1-p)}) - (1 - e^{-\sigma A}) \\ &= e^{\sigma A p/(1-p)} - 1 \end{aligned}$$

Since $r < \frac{1}{2}$, the jamming regions around v_1 and v_2 do not overlap. Using the uniformity of the network, for $i = 1, \ldots, n-1$, $E[w_i] = E[w_0]$. Thus, the end-to-end latency is,

$$E[w] = \sum_{i=0}^{n-1} E[w_i] = nE[w_1] = n(e^{\frac{\sigma A p}{1-p}} - 1).$$

■

Appendix B

In this appendix proof of Theorem 1 is presented.

Proof: In this case since the transmission range of each jammer is large, $r > \frac{1}{2}$, the jamming regions around the system nodes overlap. Let us show the event that that j^{th} link is open at the k^{th} time slot by $\tilde{l}_j(k)$. Because of the linearity of the network, all the jammers common between jamming region of the j^{th} node and any of its previous system nodes are a subset of the jammers common between the j^{th} node and the $(j-1)^{th}$ node. Hence, the probability of a link being open given that its previous links are open just depends on the last link, i.e.

$$p(\tilde{l}_j(k)|\tilde{l}_1(k), \ldots, \tilde{l}_{j-1}(k)) = p(\tilde{l}_j(k)|\tilde{l}_{j-1}(k)), \quad (1)$$

for $j = 2, \ldots, n-1$.

Now consider the link l_j and assume v_j wants to send a message to v_{j+1}. Suppose M_C denotes the number of jammers common between them and M denotes the number of jammers that just affects v_{j+1}. We want to calculate the waiting time at v_j. The probability that waiting time is zero is that all the jammers that just affect v_{j+1} are off. The reason is that zero waiting time means that the message just arrived as v_j and thus all the jammers around v_j are off. This means that all the jammers common between v_j and v_{j+1} are also off and in order to be able to send the message to v_{j+1} without delay(zero waiting time), we just need that the jammers that just affect v_{j+1} are off. Thus,

$$p(w_j = 0|M_C = m_C, M = m) = (1-p)^m$$

But if $w_j \neq 0$, the message has to wait at v_j until the next time slot that the state of jammers change. Since the state of jammers change independently from their previous states, the probability that waiting time at v_j is greater than k time slots is:

$$\begin{aligned} p(w_j > k|M_C = m_C, M = m) \\ = (1-(1-p)^m)(1-(1-p)^{m+m_C})^k \end{aligned}$$

Let us denote the area of the region that contains the jammers just affecting v_{j+1} by A and the area of the region that contains the jammers common between v_j and v_{j+1} by A_C. Using the law of total probability,

$$p(w_1 > k) = \sum_{m_C=0}^{\infty}\sum_{m=0}^{\infty}(1-(1-p)^m)(1-(1-p)^{m+m_C})^k \frac{e^{-\sigma A_C}(\sigma A_C)^{m_C}}{m_C!}\frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

Thus,

$$\begin{aligned} E[w_1] &= \sum_{k=0}^{\infty} p(w_1 > k) \\ &= \sum_{k=0}^{\infty}\sum_{m_C=0}^{\infty}\sum_{m=0}^{\infty}(1-(1-p)^m)(1-(1-p)^{m+m_C})^k \frac{e^{-\sigma A_C}(\sigma A_C)^{m_C}}{m_C!}\frac{e^{-\sigma A}(\sigma A)^m}{m!} \\ &= \sum_{m_C=0}^{\infty}\sum_{m=0}^{\infty}((1-p)^{-(m+m_C)} - (1-p)^{-m_C}) \frac{e^{-\sigma A_C}(\sigma A_C)^{m_C}}{m_C!}\frac{e^{-\sigma A}(\sigma A)^m}{m!} \\ &= e^{\frac{\sigma p(A+A_C)}{1-p}} - e^{\frac{\sigma p A_C}{1-p}} \end{aligned}$$

where $A + A_C = \pi r^2$ and A_C is the common jamming area of two neighbor nodes,

$$A_C = 2r^2\cos^{-1}(\frac{1}{\sqrt{2}r}) - \sqrt{2r^2-1}.$$

Because of the uniformity of the network, the waiting time at all nodes except v_0 is the same. The waiting time at v_0 is different because there is no node before it and thus its waiting time is equal to the waiting time at a node in the previous section. Hence, the average end-to-end waiting time is:

$$\begin{aligned} E[w] &= E[w_0] + (n-1)E[w_1] \\ &= e^{\frac{\sigma p\pi r^2}{1-p}} - 1 + (n-1)(e^{\frac{\sigma p\pi r^2}{1-p}} - e^{\frac{\sigma p A_C}{1-p}}) \end{aligned}$$

■

APPENDIX C

In this appendix our goal is to prove Theorem 2. We first investigate the waiting time at v_0. Consider the link that connects v_0 to v_1. Let us denote the process associated to this link by $l_1 = \{l_1(t)\}_{t=-\infty}^{t=\infty}$. $l_1(t) = 0$ when the link is open, i.e. all the jammers affecting this link are silent, and $l(t) \neq 0$ when the link is closed, i.e. at least one jammer affecting this link is in ON state.

Lemma 2. The link process l_1 is an alternating renewal process.

Proof: For each link, every time that the link process transitions to an open state a renewal occurs. From the memoryless property of exponential random variables, each time a link process transitions to an open state the process probabilistically starts over and hence the interarrival times are independent and identically distributed. ■

The waiting time at v_0 is denoted by w_0. Using the law of total probability, the average waiting time at v_0 can be written as,

$$E[w_0] = p(w_0 > 0)E[w_0|w_0 > 0]$$

Now suppose that the jammers are operating for a long time, e.g. their jamming process started at $t = -\infty$. Let us denote the number of jammers affecting v_1 by M. Given that $M = m$ number of jammers are in the distance less than r from v_1, the probability of zero waiting time at v_0 can be obtained as,

$$\begin{aligned} p(w_0 = 0|M = m) &= p(\text{All jammers affecting } v_1 \text{ are off}) \\ &= (p(\text{One jammers is off}))^m \\ &= \left(\frac{\mu}{\mu+\lambda}\right)^m \end{aligned} \tag{2}$$

where the second equality is from the fact that the jammers are independent and the third equality is due to [11, Theorem 3.4.4]; hence,

$$E[w_0|M = m] = \left(1 - \left(\frac{\mu}{\mu+\lambda}\right)^m\right) E[w_0|M = m, w_0 > 0] \tag{3}$$

Now suppose Z is the random variable associated with the duration of an open state of the link between v_0 and v_1 and Y is the random variable associated with the duration of a close state in the link process cycle. The following lemma characterizes $E[w_0|w_0 > 0]$ in terms of the first and the second moments of Y.

Lemma 3. The average waiting time at v_0 given that the link is closed is given by,

$$E[w_0|w_0 > 0] = \frac{E[Y^2]}{2E[Y]}$$

Proof: Let us define the random variable T as $T = 2w_0|w_0 > 0$, i.e. T denotes the length of a closed link period given that the link is currently closed. To obtain $p(T > u)$, construct a new link process l_1' from the original link process

l_1 as follows. If $Y > u$, replace it with $l'_1 = 0$ of the same length as Y, and if $Y < u$, replace it with an $l'_1 = 0$ period of length 0. Hence,

$$\begin{aligned} p(l'_1 = 0) &= p(Y > u) \\ &= p(l_1 = 0)p(Y > u|l = 0) \\ &= p(l_1 = 0)p(T > u) \end{aligned}$$

where, assuming that the jammers are operating from $t = -\infty$,

$$p(l'_1 = 0) = \frac{E[Y|Y > u]p(Y > u)}{E[Y] + E[Z]},$$

and,

$$p(l_1 = 0) = \frac{E[Y]}{E[Y] + E[Z]}.$$

Hence,

$$p(T > u) = \frac{E[Y|Y > u]p(Y > u)}{E[Y]}$$

Consequently, $E[T]$ can be obtained as,

$$\begin{aligned} E[T] &= \int_0^\infty p(T > u)du \\ &= \int_0^\infty \frac{E[Y|Y > u]p(Y > u)}{E[Y]}du \\ &= \frac{1}{E[Y]}\int_0^\infty \int_u^\infty y f_Y(y)dydu \\ &= \frac{1}{E[Y]}\int_0^\infty \int_0^y y f_Y(y)dudy \\ &= \frac{1}{E[Y]}\int_0^\infty y^2 f_Y(y)dy = \frac{E[Y^2]}{E[Y]}. \end{aligned}$$

Thus, from $E[T] = 2E[w_0|w_0 > 0]$, the lemma follows. ■

From Lemma 3, to obtain the average waiting time at v_0, we need $E[Y]$ and $E[Y^2]$. Considering the link process l_1 is running for a long time, from [11, Theorem 3.4.4] we obtain that,

$$p(w_0 = 0) = \frac{E[Z]}{E[Y] + E[Z]} \tag{4}$$

Given that m jammers affecting v_1, since Z is the minimum of m exponential random variables, $Z \sim \exp(m\lambda)$. Thus, from (2) and (4),

$$E[Y] = \frac{1}{m\lambda}\left[\left(1 + \frac{\lambda}{\mu}\right)^m - 1\right] \tag{5}$$

The second moment of the duration of a close state in the link process cycle, Y, is characterized in the following lemma.

Lemma 4. For the case that $\mu = \lambda$, given that $M = m$, we have,

$$E[Y^2] = \frac{1}{(m\lambda)^2}$$

Proof: To find $E[Y^2]$ given that m jammers affecting v_1, we model the link process by a birth-and-death process shown in Figure 2. When the link process is at state i, it means that i out of m jammers are on. The link is open ($l_1 = 0$) when all the jammers are off and the link is closed otherwise. The transition probabilities are as follows:

$$\begin{cases} p_{i,i+1} = \frac{m-i}{m}, & 0 \le i < m \\ p_{i,i-1} = \frac{i}{m}, & 0 < i \le m \\ p_{i,j} = 0, & |j - i| > 1 \end{cases} \tag{6}$$

and the birth rate at state i is $\lambda_i = (m-i)\lambda$ and the death rate is $\mu_i = i\mu$. Let us denote the time between successive visits to state 0 (link open) by X_0; thus, $X_0 = Y + Z$. Since Y and Z are independent, we have,

$$E[Y^2] = E[X_0^2] - E[Z^2] - 2E[Y]E[Z] \tag{7}$$

Let us denote P_i as the limiting probability of being in state i, i.e. $P_i = \lim_{t\to\infty} P_{ji}(t)$,

$$P_i = \frac{\binom{m}{i}\alpha^i}{(1+\alpha)^m},$$

where $\alpha = \frac{\lambda}{\mu}$. For a continuous time Markov chain, the second moment of the return time to a state can be calculated as [11],

$$E[X_0^2] = 2E[X_0]\sum_{i=0}^{\infty} P_i E[X_i] \tag{8}$$

where $E[X_i]$ is the mean sojourn time from state k to state 0 and satisfies the following set of linear equations:

$$E[X_i] = \frac{1}{\nu_i} + \sum_{j=1}^{m} P_{ij}E[X_j], \quad i \ge 0, \tag{9}$$

where ν_i is the rate that the process leaves state i, i.e. $\nu_i = \lambda_i + \mu_i$. Thus, by substituting P_{ij}s from (6) in the summation in the right side of (8), we obtain that,

$$\begin{aligned} &\sum_{i=0}^{m} P_i E[X_i] \\ &= \sum_{i=0}^{m}\frac{P_i}{\nu_i} + \sum_{i=0}^{m-1} P_i\lambda_i E[X_{i+1}] + \sum_{i=2}^{m} P_i\mu_i E[X_{i-1}] \\ &= \sum_{i=0}^{m}\frac{P_i}{\nu_i} + \sum_{i=0}^{m-1} P_i(m-i)\lambda E[X_{i+1}] + \sum_{i=2}^{m} P_i i\mu E[X_{i-1}] \\ &= \sum_{i=0}^{m}\frac{P_i}{\nu_i} + \sum_{i=0}^{m-1} P_{i+1}(i+1)\mu E[X_{i+1}] \\ &\qquad + \sum_{i=2}^{m} P_{i-1}(m-i+1)\lambda E[X_{i-1}] \\ &= \sum_{i=0}^{m}\frac{P_i}{\nu_i} + \sum_{i=1}^{m} \mu i P_i E[X_i] + \sum_{i=1}^{m} \lambda(m-i)P_i E[X_i] \\ &= \sum_{i=0}^{m}\frac{P_i}{\nu_i} + \sum_{i=1}^{m}\{m\lambda P_i E[X_i] + i(\mu-\lambda)P_i E[X_i]\} \end{aligned}$$

For simplicity we assume that $\mu = \lambda$. Hence,

$$\begin{aligned} \sum_{i=0}^{m} P_i E[X_i] &= \frac{1}{1-m\lambda}\left(\sum_{i=0}^{m}\frac{P_i}{\nu_i} - m\lambda P_0 X_0\right) \\ &= \frac{1}{1-m\lambda}\left(\frac{1}{m\lambda} - 1\right) = \frac{1}{m\lambda} \end{aligned}$$

where the second equality is due to the fact that $\sum_{i=0}^{m} P_i = 1$ and since $\lambda = \mu$, $\nu_i = m\lambda$.

Consequently, the second moment of the return time is,

$$E[X_0^2] = 2E[X_0]\sum_{i=0}^{\infty} P_i E[X_i] = \frac{2}{\nu_0 P_0 m\lambda} = \frac{2^{m+1}}{(m\lambda)^2}$$

Hence,

$$E[Y^2] = E[X_0^2] - E[Z^2] - 2E[Y]E[Z] = \frac{2^{m+1}}{(m\lambda)^2} - \frac{1}{(m\lambda)^2} - 2\left(\frac{2^m-1}{m\lambda}\right)\left(\frac{1}{m\lambda}\right) = \frac{1}{(m\lambda)^2}$$

■

Now we can prove Theorem 2 and find the average end-to-end latency of the network.

Proof: From (3), given that m jammers are affecting v_1, the average waiting time at v_0 is,

$$E[w_0|M=m] = \left(1 - \left(\frac{\mu}{\mu+\lambda}\right)^m\right) E[w_0|w_0>0] \quad (10)$$

Using Lemma 3,

$$E[w_0|M=m] = \left(1 - \left(\frac{\mu}{\mu+\lambda}\right)^m\right) \frac{E[Y^2]}{2E[Y]} \quad (11)$$

By using (5) and Lemma 4, for the case of $\mu = \lambda$ we obtain that,

$$E[w_0|M=m] = \left(1 - \frac{1}{2^m}\right)\frac{E[Y^2]}{2E[Y]} = \left(1 - \frac{1}{2^m}\right)\frac{m\lambda}{2(m\lambda)^2(2^m-1)} = \frac{1}{2^{(m+1)}(m\lambda)}$$

Hence, using the law of total probability on the number of jammers around v_1,

$$E[w_0] = \sum_{m=1}^{\infty} E[w_0|M=m]\frac{e^{-\sigma A}(\sigma A)^m}{m!} = \frac{e^{-\sigma A}}{2\lambda}\sum_{m=1}^{\infty}\frac{(\sigma A/2)^m}{m.m!}$$

$$\sum_{m=1}^{\infty}\frac{1}{2^{(m+1)}(m\lambda)}\frac{e^{-\sigma A}(\sigma A)^m}{m!} = \frac{e^{-\sigma A}}{2\lambda}(\mathrm{Ei}(\sigma A/2) - \ln(\sigma A/2)),$$

where Ei(.) is the exponential integral.

Since the jammers affecting each node affects just one node ($r < 1/2$), and because of the uniformity of the network, the average waiting times at all nodes are the same. Hence, the end-to-end latency is,

$$E[w] = \sum_{k=0}^{n-1} E[w_k] = nE[w_0] = \frac{ne^{-\sigma A}}{2\lambda}(\mathrm{Ei}(\sigma A/2) - \ln(\sigma A/2)).$$

■

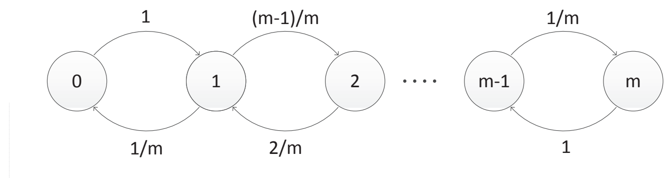

Fig. 2. Equivalent birth-and-death process of the link l_1 in the presence of m independent on-off jammers. When the process is in state i, it means i jammers are on and the rest are off. Thus, the link is open if the process is in state 0 (all the jammers are off). The birth rate is $\lambda_i = (m-i)\lambda$ and the death rate is $\mu_i = i\mu$.

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