

Logic Systems

CPSC 433: Artificial Intelligence Fall 2025

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Propositional Logic Example

Example

- Represent the following statements in propositional logic:
 - A Porsche is a black car.
 - Black cars are fast cars.
 - Bad cars are slow cars.
- Home exercise:
Show that the following statement is a logical consequence of the statements above:
 - A Porsche is a good car.

Example

- Represent the following statements in propositional logic:
 - A Porsche is a black car. $porsche \wedge black$
 - Black cars are fast cars. $black \rightarrow fast$
 - Bad cars are slow cars. $bad \rightarrow \neg fast$
- Home exercise:
Show that the following statement is a logical consequence of the statements above:
 - A Porsche is a good car.

Example

- $porsche \wedge black$
- $black \rightarrow fast$
- $bad \rightarrow \neg fast$
- A Porsche is a good car.

Example

- $p \wedge bl$
- $bl \rightarrow f$
- $b \rightarrow \neg f$
- A Porsche is a good car.

Example

- $p \wedge bl$
- $bl \rightarrow f$
- $b \rightarrow \neg f$
- $p \wedge \neg b$

Example

- $p \wedge bl$
- $bl \rightarrow f$
- $b \rightarrow \neg f$
- $\neg(p \wedge \neg b) = \neg p \vee b$

Example

- $p \wedge bl$
- $bl \rightarrow f$
- $b \rightarrow \neg f$
- $\neg p \vee b$

Example

- p
- bl
- $bl \rightarrow f$
- $b \rightarrow \neg f$
- $\neg p \vee b$

Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$

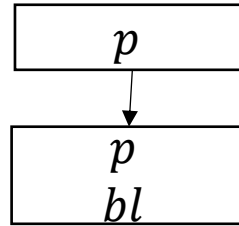
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$

p

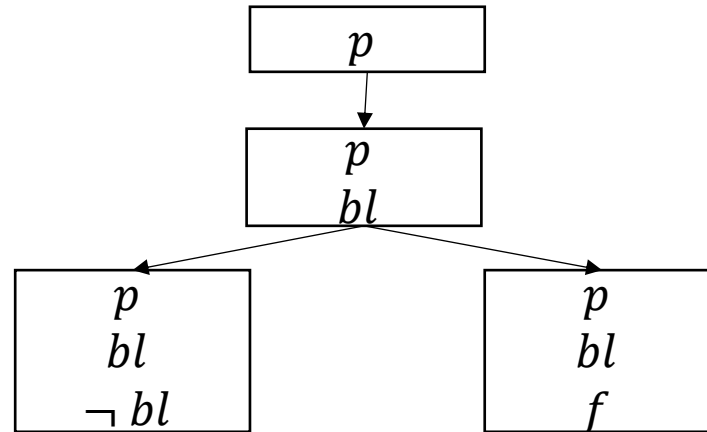
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



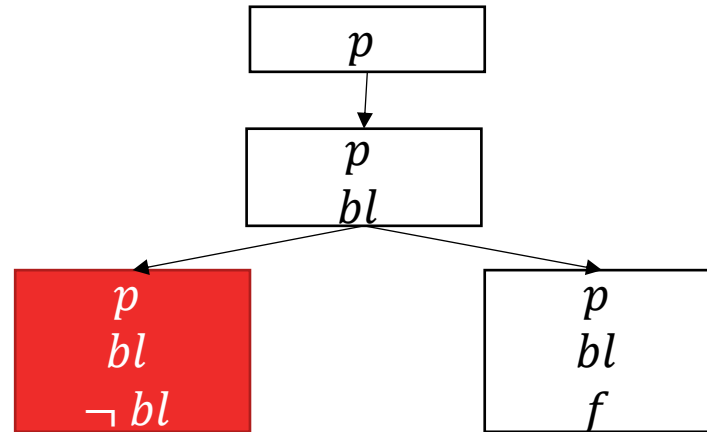
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



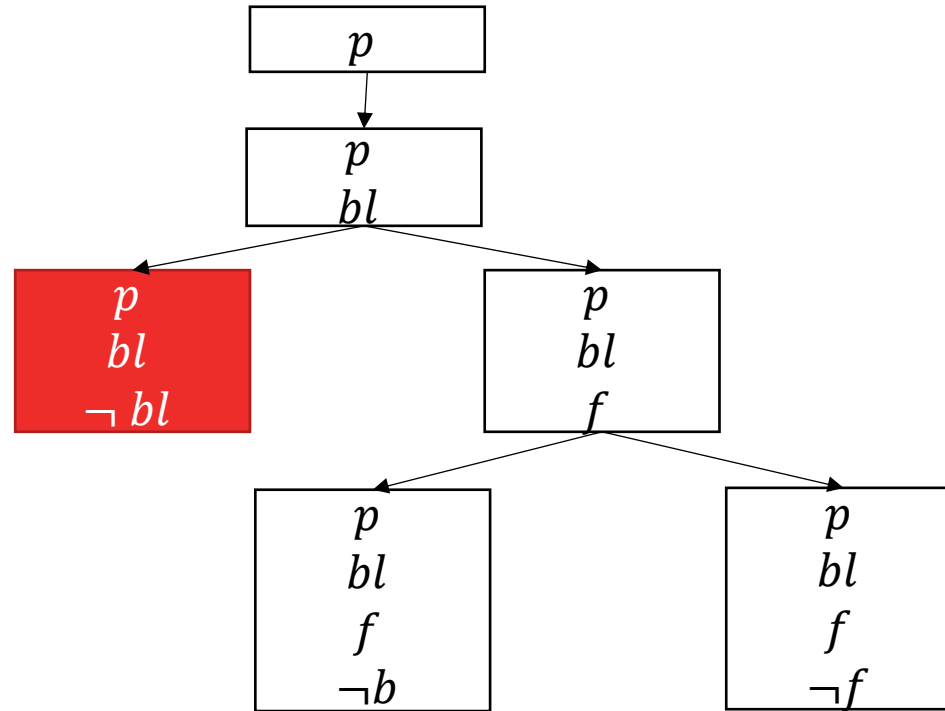
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



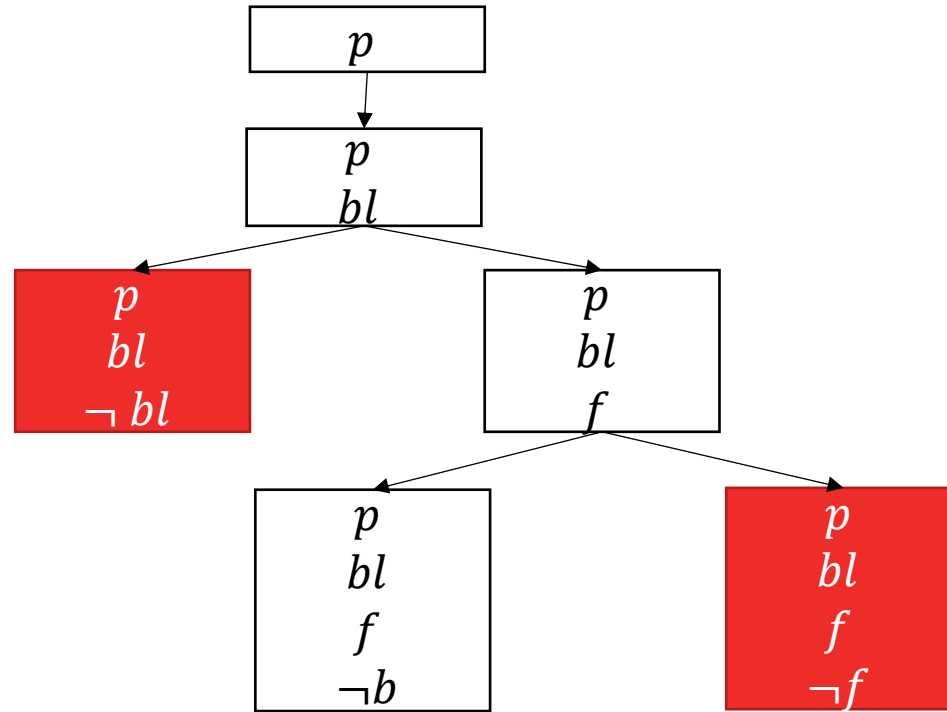
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



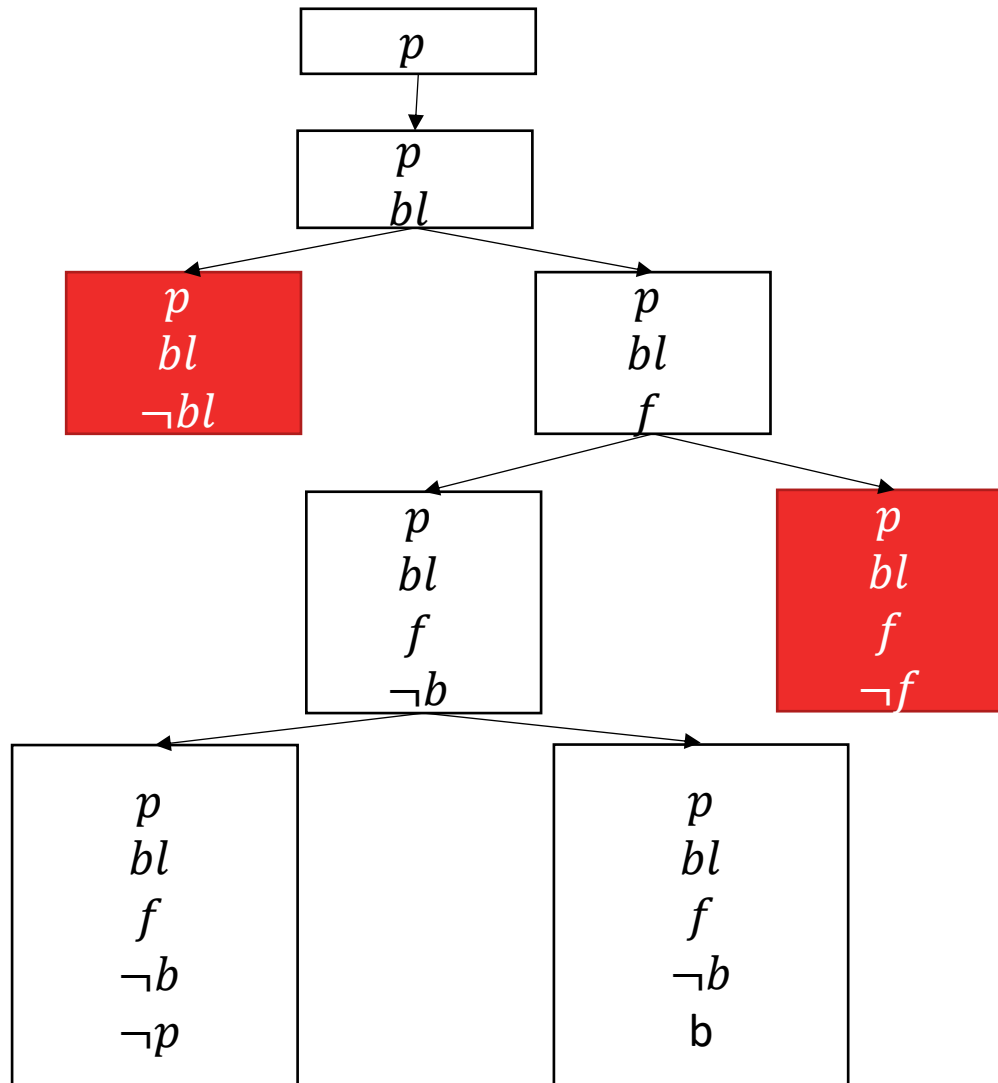
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



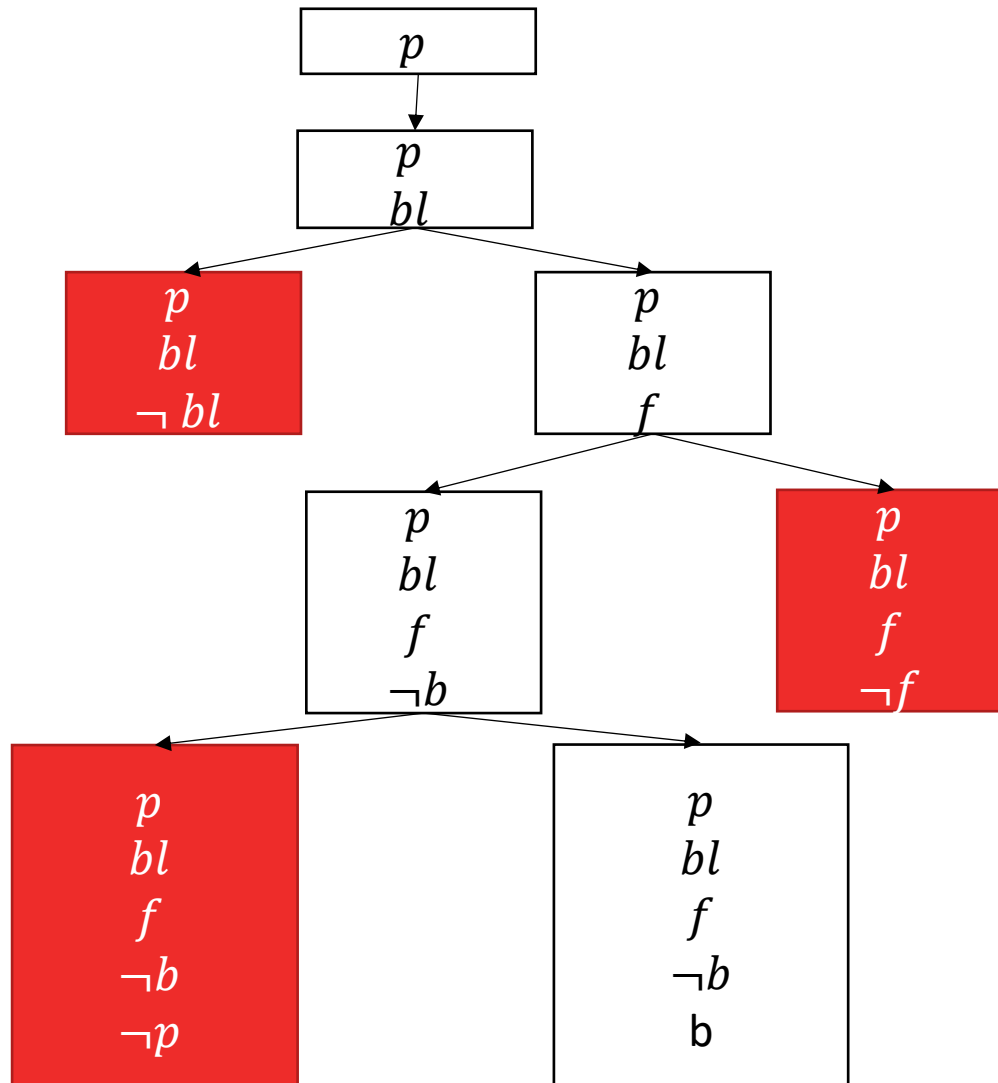
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



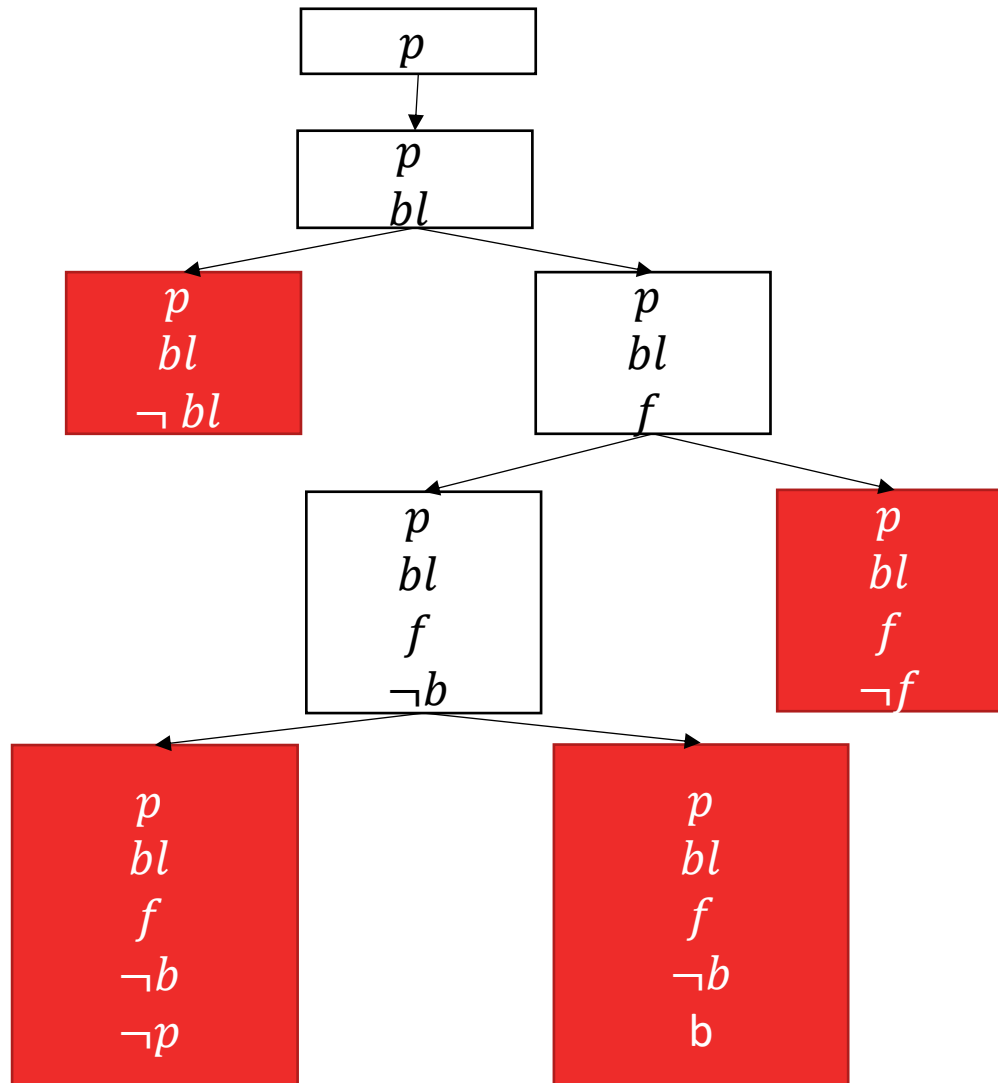
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



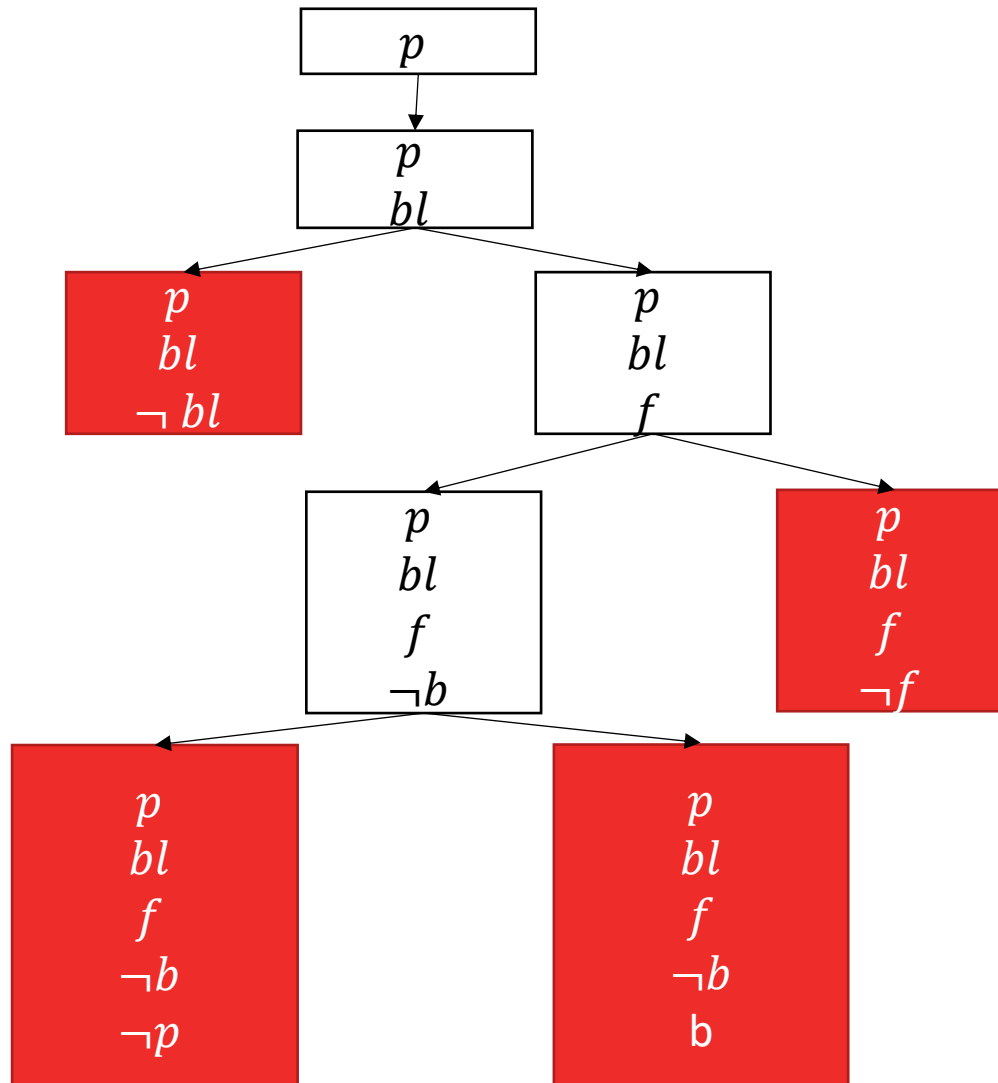
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



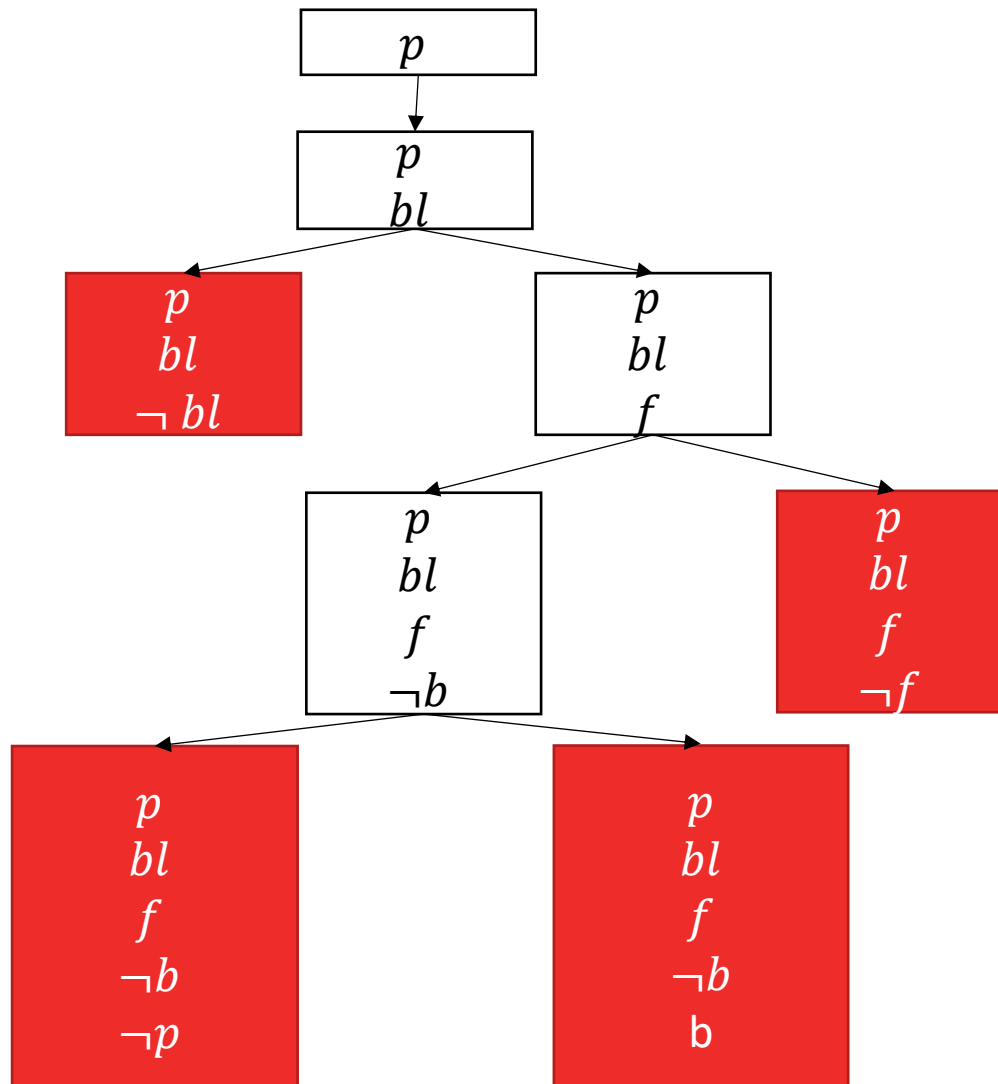
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$



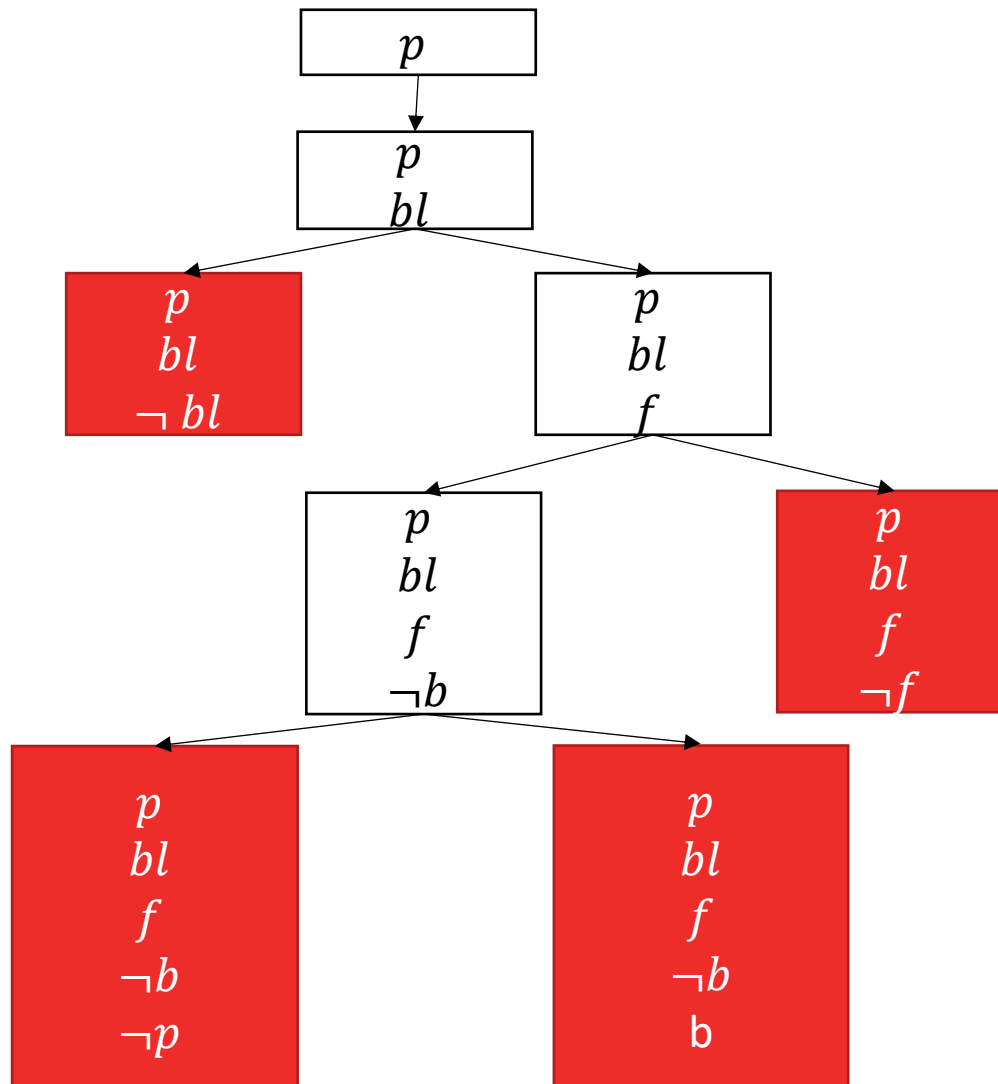
Example

- p
- bl
- $\neg bl \vee f$
- $\neg b \vee \neg f$
- $\neg p \vee b$
- $\neg(\neg p \vee b)$



Example

- p
 - bl
 - $\neg bl \vee f$
 - $\neg b \vee \neg f$
 - $\neg p \vee b$
-
- $p \wedge \neg b$
 - A Porsche is a good car.



First-Order Logic Example

Example

- Use PL1 for the example for propositional logic (2!)
- Home exercise:
Show that the statements
 - Everyone who lies is a bad person
 - I know a politician who liesimplies the statement
 - There is a politician who is a bad person

Example

- Represent the following statements in propositional logic:
 - A Porsche is a black car. $\text{black}(p)$
 - Black cars are fast cars. for all x $\text{black}(x) \rightarrow \text{fast}(x)$
 - Bad cars are slow cars. for all x $\text{bad}(x) \rightarrow \text{not fast}(x)$
- Home exercise:
Show that the following statement is a logical consequence of the statements above:
 - A Porsche is a good car. $\text{good}(p)$

Example

- Represent the following statements in propositional logic:
 - A Porsche is a black car. $\text{black}(p)$
 - Black cars are fast cars. for all x $\text{black}(x) \rightarrow \text{fast}(x)$
 - Bad cars are slow cars. for all x $\text{bad}(x) \rightarrow \text{not fast}(x)$
- Home exercise:
Show that the following statement is a logical consequence of the statements above:
 - A Porsche is a good car. $\text{not bad}(p)$

Example

- $\text{black}(p)$
- for all x $\text{black}(x) \rightarrow \text{fast}(x)$
- for all x $\text{bad}(x) \rightarrow \text{not fast}(x)$
- $\text{not bad}(p)$

Example

- $\text{black}(p)$
- for all x $\text{black}(x) \rightarrow \text{fast}(x)$
- for all x $\text{bad}(x) \rightarrow \text{not fast}(x)$
- $\text{not bad}(p)$

Example

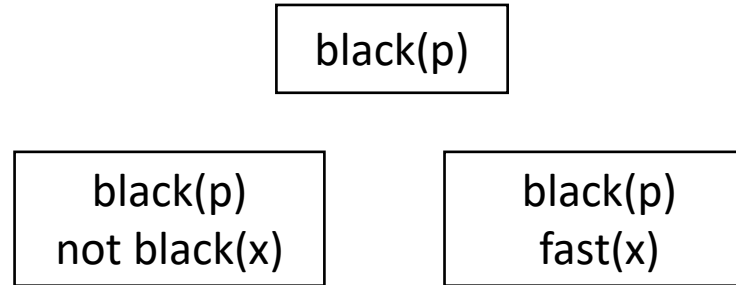
- $\text{black}(p)$
- for all x $\text{black}(x) \rightarrow \text{fast}(x)$
- for all x $\text{bad}(x) \rightarrow \text{not fast}(x)$
- $\text{bad}(p)$

Example

- $\text{black}(p)$
- $\text{not black}(x) \text{ or fast}(x)$
- $\text{not bad}(x) \text{ or not fast}(x)$
- $\text{bad}(p)$

Example

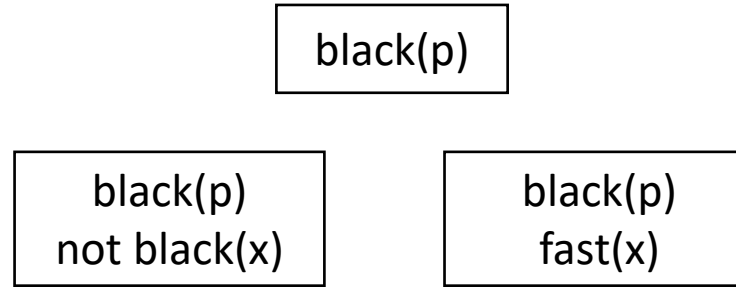
- $\text{black}(p)$
- $\text{not black}(x) \text{ or fast}(x)$
- $\text{not bad}(x) \text{ or not fast}(x)$
- $\text{bad}(p)$



Example

- $\text{black}(p)$
- $\text{not black}(x) \text{ or fast}(x)$
- $\text{not bad}(x) \text{ or not fast}(x)$
- $\text{bad}(p)$

$$\text{mgu} = \{x = p\}$$



Example

- $\text{black}(p)$
- $\text{not black}(x) \text{ or fast}(x)$
- $\text{not bad}(x) \text{ or not fast}(x)$
- $\text{bad}(p)$

$$\text{mgu} = \{x = p\}$$

$\text{black}(p)$

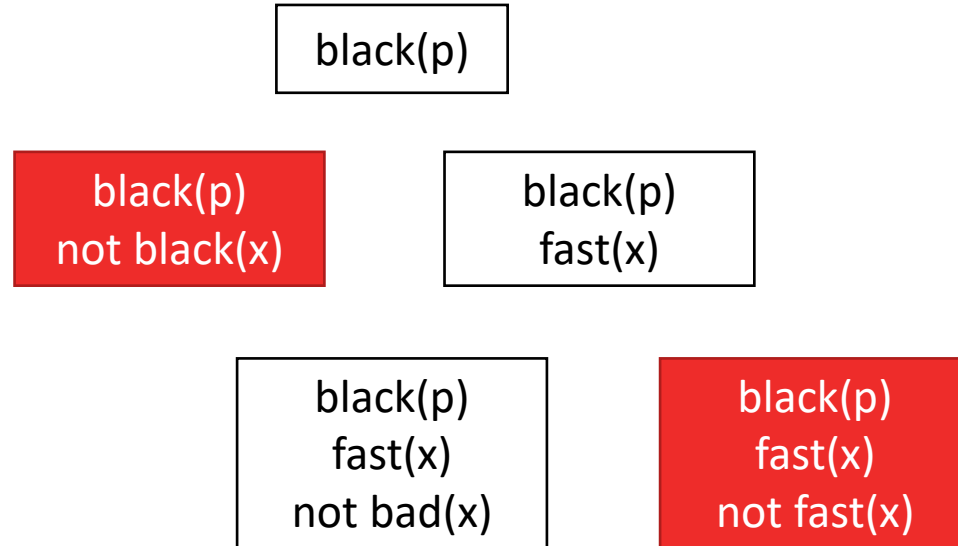
$\text{black}(p)$
 $\text{not black}(x)$

$\text{black}(p)$
 $\text{fast}(x)$

Example

- $\text{black}(p)$
- $\text{not black}(x)$ or $\text{fast}(x)$
- $\text{not bad}(x)$ or $\text{not fast}(x)$
- $\text{bad}(p)$

$$\text{mgu} = \{x = p\}$$



Example

- black(p)
- not black(x) or fast(x)
- not bad(x) or not fast(x)
- bad(p)

$\text{mgu} = \{x = p\}$

black(p)

black(p)
not black(x)

black(p)
fast(x)

black(p)
fast(x)
not bad(x)

black(p)
fast(x)
not fast(x)

black(p)
fast(x)
not bad(x)
bad(p)

Example

- black(p)
- not black(x) or fast(x)
- not bad(x) or not fast(x)
- bad(p)

$$\text{mgu} = \{x = p\}$$

black(p)

black(p)
not black(x)

black(p)
fast(x)

black(p)
fast(x)
not bad(x)

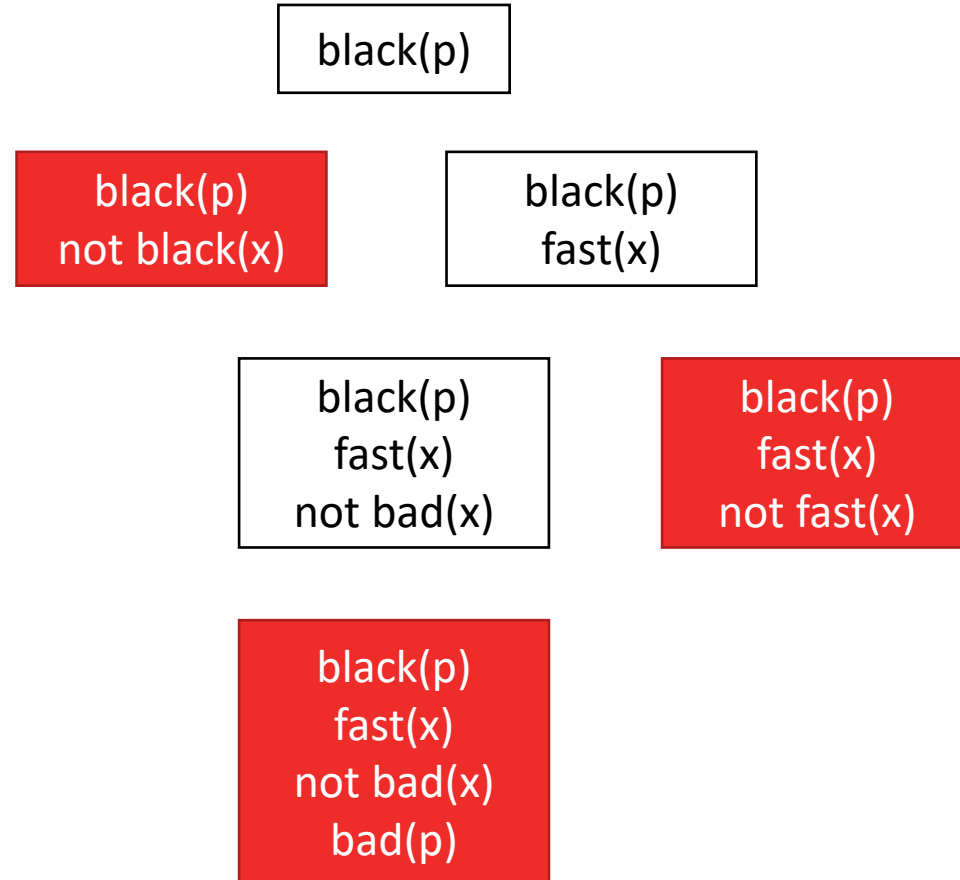
black(p)
fast(x)
not fast(x)

black(p)
fast(x)
not bad(x)
bad(p)

Example

- black(p)
- not black(x) or fast(x)
- not bad(x) or not fast(x)
- not bad(p)

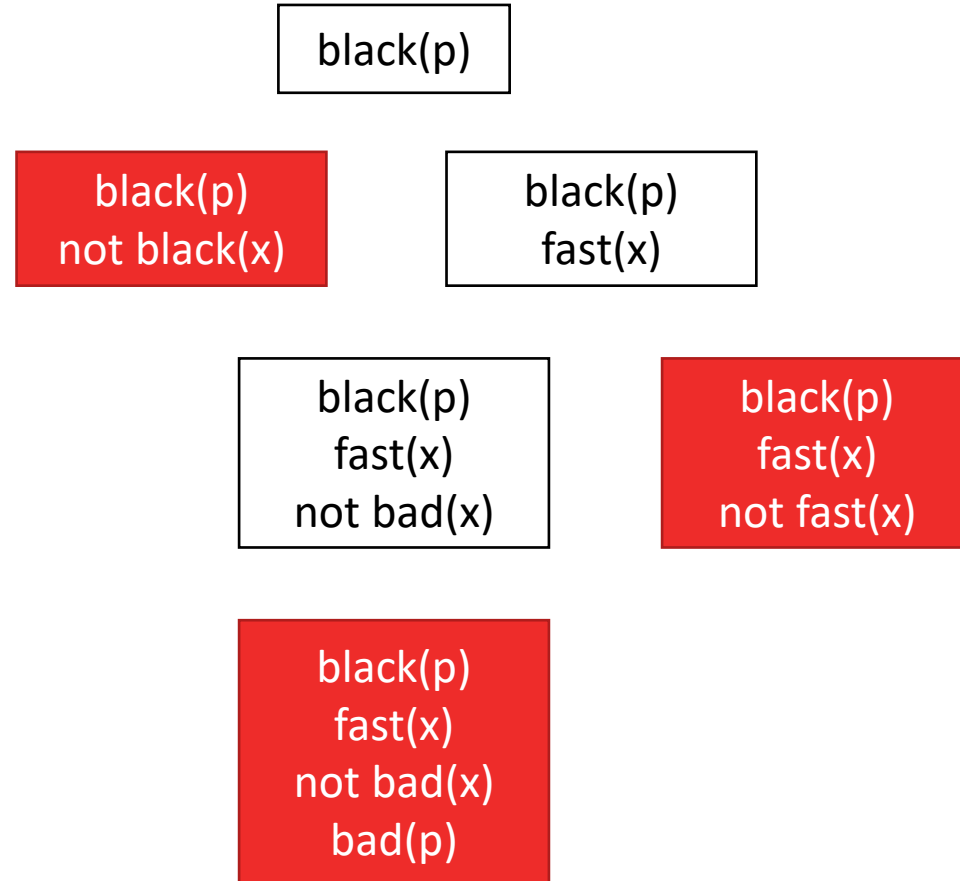
$$\text{mgu} = \{x = p\}$$



Example

- black(p)
- not black(x) or fast(x)
- not bad(x) or not fast(x)
- A Porsche is a good car.

$$\text{mgu} = \{x = p\}$$



Onward to ... rule systems

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