MAS – Game Theory & Social Welfare

CPSC 383: Explorations in Artificial Intelligence and Machine Learning
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Game Theory

Game Theory

Game theory: theory of strategic decision making.

- players each taking into account how other players may act
- Used in agent design and mechanism design

1. It can be a cooperative game where a binding agreement between agents exists enabling robust cooperation.
2. non-cooperative game: no central agreement and no guarantee of cooperation

Non-Cooperative Game Theory

Games with a single move: Normal form games

- All players take action simultaneously
- No player has knowledge of the other players' choices
- Defined by 3 components
  - Players
  - Actions
  - Payoff function: utility to each player for each combination of actions by all the players (payoff matrix)

	Player 1: Choice 1	Player 1: Choice 2
Player 2: Choice 1		
Player 2: Choice 2		

Non-Cooperative Game Theory

Games with a single move: Normal form games

- The role of solution concepts in game theory is to try to make reasoning precise
- **A pure strategy** is a deterministic policy; for a single-move game, a pure strategy is just a single action.
- **Mixed strategy:** a randomized policy that selects actions according to a probability distribution.
- A **strategy profile** is an assignment of a strategy to each player

Non-Cooperative Game Theory

Two-finger Morra game

- two players, 1 and 2
- simultaneously display one or two fingers.
- Let the total number of fingers displayed be *f*.
  - If *f* is odd, *O* collects *f* dollars from *E*;
  - and if *f* is even, *E* collects *f* dollars from *O*.
- The payoff matrix for two-finger Morra is as follows:
  - (Player 1 payoff, Player 2 payoff)

	Player 1: one finger	Player 1: two fingers
Player 2: one finger	(2,-2)	(-3,3)
Player 2: two fingers	(-3,3)	(4,-4)

Non-Cooperative Game Theory

Two-finger Morra game

- Player 1 -> [-3,-3, 2, 4]
- Player 2 -> [-4, -2, 3, 3]

	Player 1: one finger	Player 1: two fingers
Player 2: one finger	(2,-2)	(-3,3)
Player 2: two fingers	(-3,3)	(4,-4)

Non-Cooperative Game Theory

Two-finger Morra game

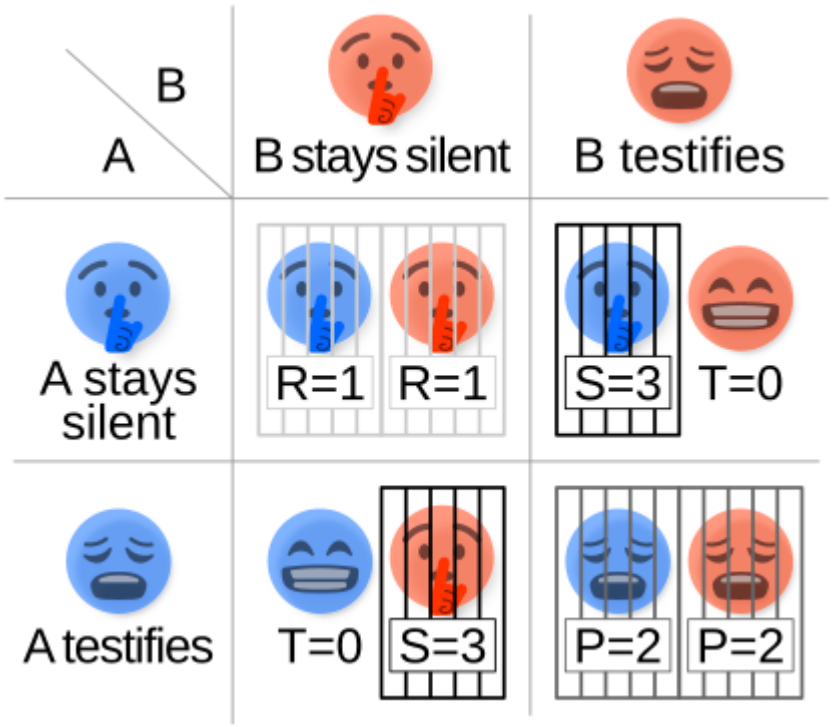
- Player 1 -> [-3,-3, 2, 4]
  - When p1 loses -> lose -3
  - Pick better pair with pay-off where I make more (4) based on p2 choice unknown
- Player 2 -> [-4, -2, 3, 3]
  - When p2 wins -> wins 3
  - Pick better pair with loss where I lose less (-2) based on p1 choice unknown

- Result would be p1 loses 3, and p2 gains 3

	Player 1: one finger	**Player 1: two fingers**
Player 2: one finger	(2,**-2**)	(**-3**,**3**)
Player 2: two fingers	(-3,3)	(**4**,-4)

Non-Cooperative Game Theory

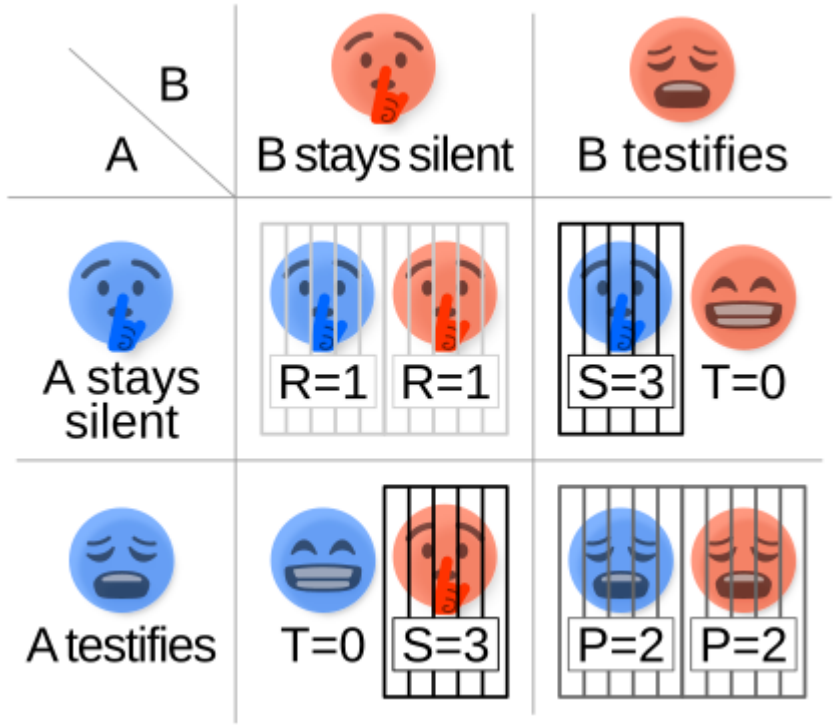
- Two prisoners are separated into individual rooms and cannot communicate with each other
- What should each choose to do?
  - Testify (defect)?
  - Stay silent?



Non-Cooperative Game Theory

- Two prisoners are separated into individual rooms and cannot communicate with each other.

1. It is assumed that both prisoners understand the nature of the game,
2. have no loyalty to each other,
3. and will have no opportunity for retribution or reward outside of the game.



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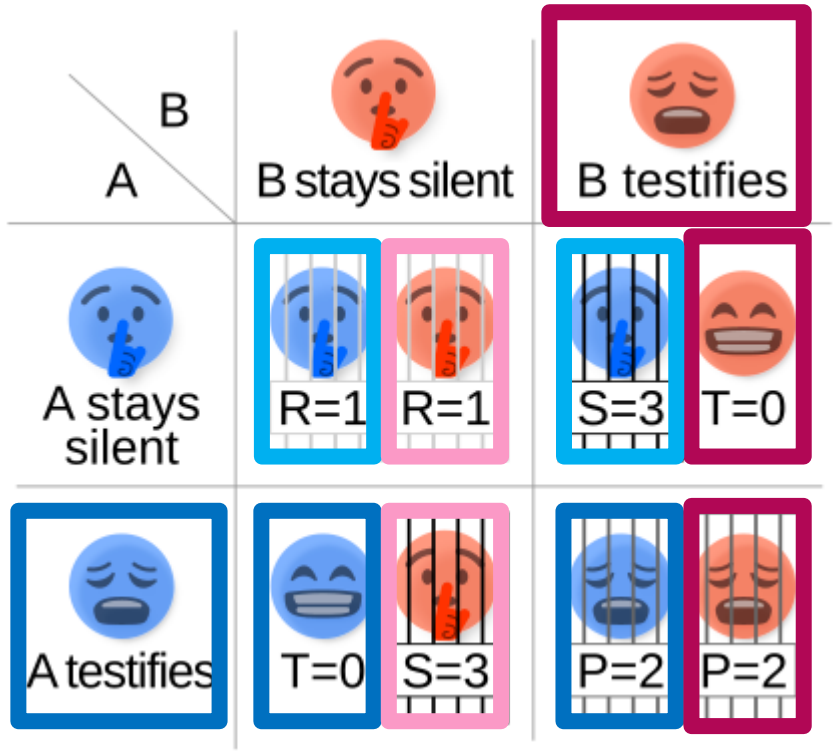
Non-Cooperative Game Theory

- Two prisoners are separated into individual rooms and cannot communicate with each other.

1. Defection always results in a better payoff than cooperation, so it is a strictly **dominant strategy** for both players.
2. Mutual defection is the only strong **Nash equilibrium** in the game.
    1. situation where no player could gain by changing their own strategy (holding all other players' strategies fixed)
3. Since the collectively ideal result of mutual cooperation is irrational from a self-interested standpoint, this Nash equilibrium is **not Pareto efficient**.



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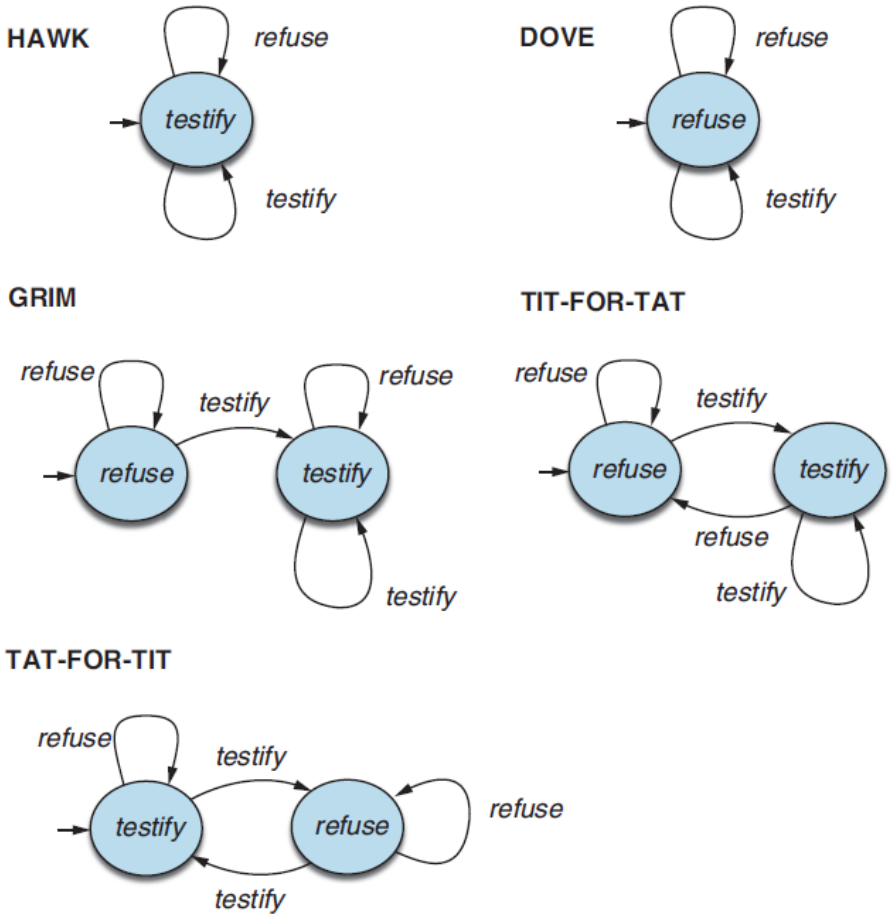
Repeated games

Some common, colorfully named finite-state machine strategies for the infinitely repeated prisoner's dilemma

Tit-for-Tat strategy simply copy whatever its counterpart did on the previous round.

GRIM strategy is somewhat similar to TIT-FOR-TAT, but with one important difference: if ever its counterpart plays its dominant action, then it essentially turns into HAWK



Social Welfare

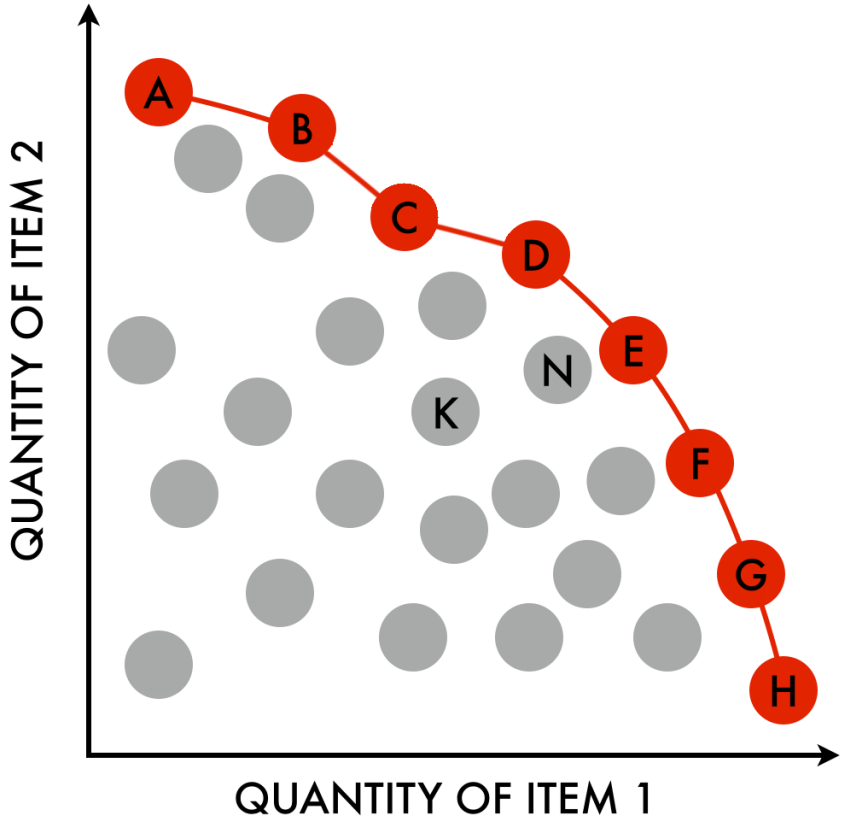
Social Welfare

Want to choose the best overall outcome—the outcome that would be best for society as a whole avoid outcomes that waste utility

Pareto optimality: there is no other outcome that would make one player better off without making someone else worse off

Utilitarian social welfare is a measure of how good an outcome is in the aggregate

Egalitarian social welfare: maximize expected utility of the worst-off member of society



Social Welfare: Computing equilibria

Myopic best response or iterated best response

1. start by choosing a strategy profile at random; then,
2. if some player is not playing their optimal choice given the choices of others, flip their choice to an optimal one, and repeat the process

The process will converge if it leads to a strategy profile in which every player is making an optimal choice, given the choices of the others—**a Nash equilibrium**

Next…making collective decisions

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