RESEARCH NOTES ON FUZZY REASONING

B.R.Gaines

(1) Fuzzy Logic and Fuzzy Reasoning

There appears to be an important gap between the so-called "fuzzy logics" studied by many authors (e.g. Lee 1972 - JACM, Bellman and Giertz 1973, Inf. Sc.) and the "fuzzy reasoning" developed by Zadeh (Berkeley reports) which is not noted in the literature. The standard "fuzzy logic" is just a multi-valued logic of the family described by Rescher (1969 book). Most authors probably regard it as conjunction/disjunction/negation derived from fuzzy set membership considerations completely analogous to the derivation of Boolean algebra from classical set theory. Five points seem to have been missed:

(1.1) The definition of implication is open - it is not determined by the other connectives. For example, the natural definition of negation in terms of implication:

Def. $\bar{A}$: $A \supset F$

gives "fuzzy negation" ($u(\bar{A}) = 1-u(A)$) for both VSS implication ($u(A \supset B) = \max(1-u(A),\ u(B))$) and Lukasiewicz implication ($u(A \supset B) = \min(1,\ 1-u(A)+u(B))$).

Question 1 How many of Rescher's (see also Rosser and Turquette et al) logics have (or, if non-truth-functional, can be restricted to have) the fuzzy logic basic connectives ?

Question 2 What is the status of fuzzy negation, e.g. c.f. Gödel negation ?

(1.2) Some authors do not even consider implication - Lee assumes the PC definition:

Def. $A \supset B$: $\bar{A} \vee B$

without making this explicit. This then leads to the semantic inconsistency he notes for the relative magnitudes of the terms, but he does not follow this up to conclude that his definition is wrong, presumably because he does not realize he has one.

Question 3 What are the minimal semantic constraints upon the implication connective (see Carnap et al on confirmation theory) ?

(1.3) The "fuzzy logic" generally considered may be derived as the fuzzification of PC. This does not seem to have been stated explicitly but is probably folk lore. Such a derivation is more in the spirit of the general application of fuzzification to other mathematical structures.

(1.4) Fuzzified PC is precisely the variant standard system (VSS) described by Rescher (attributed to ?).

Question 4 Can other MV logics be derived as fuzzifications - e.g. Lukasiewicz and Godel (I doubt it: if not there may be some relationship to Dugundji's results on finite matrices) ?

(1.5) Zadeh himself does not seem to have put forward fuzzified PC (or rejected it because of the weakness of its implication connective ?). Fuzzy reasoning is primarily concerned with statements about fuzzy attributes - one possible attribute is a truth value. When Zadeh considers the truth value attribute he fuzzifies Lukasiewicz MV logic not PC.

Question 5 Why choose Lukasiewicz logic (Zadeh does not seem to give a justification) ?

(2) Fuzzified Lukasiewicz Logic (LL)

The similarity between the fuzzy set operations on degrees of membership and the LL basic connectives needs investigation (Quest. 4 and 5). There are semantic constraints upon the form of function that maps degree of membership onto truth value. These constraints should be made explicit. The logic without the constraints offers an apparent freedom that should be removed - this will itself lead to a different formulation.

Question 6 Given a family of functions on the unit interval (truth value → membership) and the operations of fuzzy sets theory and LL (or various subsets) what is the space of functions generated (and the converse of deriving a basis for a given space given the operations) ?

Question 7 What are the semantic constraints upon a basis ?

(3) Interaction of Truth Values with Fuzzy Statements

We may apply (essentially metalinguistic ?) statements about fuzzy truth values to statements about fuzzy attributes and there is the possibility of interplay between them. For example, what is the relationship between 'John is tall is very true' and 'John is very tall is true', or 'Mary is fat is more or less true' and 'Mary is more or less fat'. There is scope for interplay but no obvious rule. Notes:

(3.1) One must beware of unnatural examples and watch for the possibility of multiple interpretations - e.g. it does not seem to be meaningful to consider statements such as 'John is tall is .7 true', and 'John is tall is very true' could mean that he is so tall no one could disagree (in which case 'very tall' assumes a higher truth value) or that he is precisely tall, neither 'more or less tall', nor 'very tall' (in which case 'very tall' will assume a lower truth value).

(3.2) Ambiguities can only be resolved by accepting that the role of language is communication and that the same statement may have entirely different meanings for different recipients. The use of this feature of language is itself a major linguistic skill (essential to politicians).

(3.3) Our usage of words such as 'truth' and 'false' may be more related to 'reasonableness' and 'unreasonableness' than logical truth, i.e. a statement is very true because it is a reasonable way of expressing something that will create a very true impression of the state of affairs in the mind of the recipient. Thus 'John is tall is very true' would mean that it is the most reasonable statement to make about John's height. If he turns out to be 7 foot tall, you say 'but he is extremely tall, very very tall' and feel that you have been misled, i.e. 'John is tall' is 'not very true', but 'John is extremely tall' is very true.

This model of our use of the linguistic terms true and false in the metalinguistic context (i.e. about other statements) as relating to the communication of a true impression is a useful one, probably widely valid. It resolves the conflict between a direct interpretation of degrees of membership as degrees of truth - where 'John is very tall' makes 'John is tall' very true - and what seems to be the more conventional use of statements about truth and falsity in colloquial language. It also emphasizes that the analysis of linguistic interactions must be in a context of interpersonal communication, not isolated fragments.

(3.4) The use of linguistic hedges is not only to modify meaning but also to convey the level of precision. 'John is tall', 'John is more or less tall', 'John is pretty well tall', 'I think it is very true to say that John is tall', all convey the same expectation of height but varying degrees of possible spread about it. This is why a single truth value cannot express the full semantics of a vague statement.

(4) Linguistic Approximation Stable Fuzzy Arguments

Zadeh's concept of linguistic approximation (LA) introduces an element of discontinuity into the fuzzy reasoning process. LA arises basically because the numerical manipulations of fuzzy predicates corresponding to linguistic hedges and logical operations can generate a result that cannot be represented exactly as a simply hedged linguistic truth-value - it can only be approximated by one. The effect is similar to that of quantization in analog-digital conversion and generates similar problems, i.e. it cannot be treated effectively as "noise", introduces its own coloration, and gives rise to new phenomena such as limit cycles.

The importance of linguistic approximation to a theory of fuzzy reasoning seems to have been missed despite the emphasis Zadeh places upon it. Without it any form of fuzzy logic is a variant of some formal multi-valued logic and (whilst again the fact that Zadeh uses a fuzzified Lukasiewicz logic rather than fuzzified PC seems to have been little-noted) it is presumably open to axiomatization and probably to reduction to some known structure.

With LA fuzzy logic has new properties, for example that a long chain of reasoning that is logically equivalent to a shorter chain will produce less sharp results in general.

Several questions are apparent:

Question 8 What class of operation on fuzzy variables leads to a finite set of values ? - a purely technical point reducing the need for LA.

Question 9 Does LA account for the weakness in long chains of reasoning ?

We may introduce the concept of a stable fuzzy argument which is such that if LA is applied at all or any points in the chain of reasoning the LA to the final result is unchanged. This introduces the concept of a linguistic confluence set - the set of all possible results of a chain of fuzzy reasoning when LA is applied in all possible ways.

The following results are obvious: (a) the longer the chain of argument the less stable it will be; (b) the greater the range of LA's available the more stable it will be - this corresponds to the eskimos 40 names for ice, the skilled practitioner's use of longer chains of argument, etc.

LA introduces tolerance relations on the space of functions over an interval. Can we take the logic and tolerance relation and treat it as a new logic ?

(5) Fuzzified Definitions of System Concepts

It should be possible to re-develop such concepts as stability, adaptivity and state within a framework of fuzzy reasoning. Some of the arbitrariness in current definitions should be absorbed into the fuzziness rather than left as firm but undefined decisions. The semantic constraints that mean that decisions are not completely arbitrary will appear as the order relations on fuzzy values.

(6) The Role of the Numbers

How much of the theory of fuzzy reasoning can be developed in terms of order relations on degrees of membership rather than truth values. I doubt that this has been studied in the light of Zadeh's semantics for fuzzy reasoning, e.g. with fuzzified LL.

Linguistic Approximation in an order structure would give a tolerance leading to a non-truth-functional logic. This seems a very natural structure that is worth developing.

These notes were based on discussions with Lotfi Zadeh at Berkeley in May 1975.