# Living in an Uncertain Universe*

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## 1 Introduction

It has been interesting to revisit the era when my path crossed with those of John Andreae, Joe Goguen, George Klir, Ladislav Kohout, Abe Mamdani, Gordon Pask, Ted Poppelbaum and Lotfi Zadeh, all precious friends and colleagues, some of whom are, unfortunately, no longer with us; their ideas live on and flourish as a continuing inspiration. I was tempted to entitle this article after Gurdieff, *Meetings with Remarkable People*, and that will be its focus, particularly the background to the initial development of fuzzy controllers. There have been studies analyzing their history and the basis of their success—this article provides some details from my personal experience.

## 2 My Background

My path began in my schooldays when I was an electronics, mathematics and philosophy hobbiest, having been entranced by Wiener's *I am a Mathematician*, Hardy and Littlewood's *Pure Mathematics*, Bertrand Russell's *Principles of Mathematics*, Kant's *Critique of Pure Reason* and Hegel's *Science of Logic*. Logic and mathematics were ways of modelling and understanding the world, and electronics a way of influencing and controlling it. I saw everything from cosmology and society to the world of ideas as a cybernetic system with hierarchies of feedback loops evolving through temporary equilibria punctuated by periods of chaos; I still do.

The mathematics I valued was algebra and logic. I saw arithmetic as an over-specified algebraic structure, notions of continua and infinity as useful fictions for modelling indefinitely extensible algebraic structures, and the major mathematical modelling task as being the search for underlying generative systems that were logical, concise and complete; I still do.

I spent a year at ITT's semiconductor research laboratories at Footscray before going up to study mathematics at Trinity, Cambridge in 1959. I saw myself as an electronics engineer, quantum physics as the foundation of electronics and mathematics as the foundation of physics. I took with me some of the semiconductor devices I had fabricated at ITT and investigated who might find them useful; this earned me a place in Richard Gregory's cognitive psychology laboratory as his electronics technician.

## 3 Learning Machines at ITT

I saw an advertisement in *Nature* for positions in a new *Learning Machines* project at *Standard Telecommunication Laboratory* (STL), ITT's Research Laboratories at Harlow, specifying qualifications in topology, cybernetics, neural networks, and other topics that interested me, and contacted the Project Leader, John Andreae. During my years at Cambridge I spent my vacations working with John and his team, David Hill, Owen Morgan and Peter Joyce, eventually taking over his position part-time when he left to take up a Chair at the University of Canterbury, New Zealand.

*Chapter for R. Seising, E. Trillas, C. Termini (eds.) On Fuzziness: A Homage to Lotfi Zadeh, Springer, 2012

Figure 1: Three pioneers of computational intelligence: from left to right, John Andreae (learning machines), David Hill (speech recognition), Ted Poppelbaum (stochastic computing in pattern recognition), IFIP 1968, Edinburgh

I first came across the work of Lotfi Zadeh whilst at ITT. Andreae's STeLLA, the STL learning automaton, digitally encoded the state space of the system to be controlled and learned to generate inputs to keep the system state in a specified region of that space; Zadeh and Desoer's (1963) book, *Linear System Theory: The State Space Approach* was the bible for that approach to system design. In a 1963 symposium on general system theory Lotfi had presented a general formulation of *The notion of state in system theory* (Zadeh, 1964) that encompassed linear systems and automata. Although he had specifically excluded stochastic and anticipative systems it was clear how to extend his abstract framework to such systems and apply them to learning controllers.

I envisioned general learning components as black boxes in system design where the designer no direct control or knowledge of their internal states, customizing them for particular tasks through external techniques: *coding* input stimuli to make learning easier; *training* them through task sequences designed to facilitate learning; and *priming* them through linguistic stimuli designed to provide an initial problem solution to be refined by experiential learning. My state spaces were those of the learning components' capabilities, my design techniques were behavioural, and I developed a theoretical framework based on Ross Ashby's (1962) algebraic formulation of Gerard Sommerhoff's (1950) model of adaptivity in living systems in *Analytical Biology* and Lotfi's formulation of adaptivity in control systems in *On the definition of adaptivity* (Zadeh, 1963).

I kept track of Lotfi's research and saw his 1965 paper on *Fuzzy sets* (Zadeh, 1965) as an interesting engineering application of set theory based on Łukasiewiz infinite-valued logic (Hay, 1963). I was interested in that logic as Moh Shaw-Kwei (1954) had speculated the axiom of comprehension might not be subject to Russell's paradox in the corresponding set theory (still open in 1965 (Chang, 1965), proved in 1979 by Richard White (1979)), a paradox that had fascinated me since I read *Principles of Mathematics*; it seemed to present a pitfall for any axiomatic system theory.

I knew that Łukasiewiz's axioms could be subsumed under axiomatic probability logics where I saw Carnap's (1950) logical interpretation, Savage's (1954) subjective one, and Shackle's (1961) partial order over possibilities, as better models of uncertainty than frequentist ones, but found nothing I could use immediately in the fuzzy sets paper. Lotfi's footnote that "the membership function can be taken to be a suitable partially ordered set" was more appealing to my algebraic frame of mind than a mapping to the numeric range [0,1]; it still is.

## 4 Experimental Psychology at Cambridge

When I graduated from the Mathematics Tripos, Richard suggested I study for a PhD with him. Oliver Zangwill, the Chair of Experimental Psychology, accepted me but said I must get a psychology degree also. I took Part II of the Psychology Tripos after a year, preparing by writing past exam papers for my tutor, Alan Watson, acquiring the necessary background from journal papers, writing an essay on a methodology for animal experiments based on my model of adaptivity, and attending lectures on topology, probability, logic and algebra to extend my mathematical proficiency.

My doctoral research was funded through a contract with the Ministry of Defence to study adaptive training of perceptual-motor skills, and I built an analog computer in order to emulate a flight simulator and collect data on the learning of pilots from RAF Oakington under different training regimes.

Figure 2: A young Brian Gaines and analog computer in his laboratory in the Department of Experimental Psychology, Cambridge,1964

My studies of human operators at Cambridge were synergistic with my studies of learning machines at STL. My paper with John for the 1966 IFAC Congress, *A learning machine in the context of the general control problem* (Gaines & Andreae, 1966), emphasized the same techniques, of *coding*, *training* and *priming*, that I was using in my human operator studies. We *coded* the inputs and outputs of the learning systems to ensure they provided a natural topology for the problem space; *trained* the systems through a task progression dynamically generated through feedback from their performances; and *primed* them by linguistically specifying behaviours that seemed, *prima facie*, to be initially useful.

The paper ends with a remark that seems prophetic for later developments in fuzzy control:

> It is customary to think of controllers in terms of optimality but, when the plant is indeterminate or time-varying and the controller itself is required to be widely applicable, such a concept loses much of its force. When fabrication and storage costs have also to be taken into account, one can only ask whether satisfactory control is possible and, if so, how much it will cost.

That captures the ethos in which John Andreae, Igor Aleksander, Abe Mamdani and I worked on learning controllers.

## 5 Stochastic and Possibilistic Computing

In Richard's laboratory I investigated stereoscopic vision with an oscilloscope I had built with two small cathode ray tubes sending separate stimuli to each eye; enabling subjects to rotate and move simulated images of 3- and 4-D skeleton cubes and hypercubes. I speculated on the neural mechanism through which disparity was used to perceive depth; it seemed to necessitate cross-correlation between the stimuli from the two eyes. I thought it might be modelled as a neural process whereby stimulus intensities encoded as the probability of a neuron firing would be multiplied and cross-correlated by a neuron firing when it received pulses from corresponding neurons in both eyes.

I implemented this notion in one of my projects for the ITT learning machine, a front-end neural net to learn more useful encodings of the input stimuli regardless of their source and nature; John named it *Gadafter*, the 'Gaines adaptive filter.' I developed an adaptive filter based on discrete logic gates with stimuli encoded as the generating probabilities of random pulse streams, and a stochastic version of the adaptive digital elements (ADDIEs) for making the weight changes in STelLA's learning protocol, leaky stochastic integrators computing running averages.

By the time I returned to the ITT Laboratories for the Summer of 1965 I had a fully developed scheme for a *stochastic analog computer* that resulted in a massive patent application (Gaines, 1966) with 54 claims and papers in *Electronics* (Gaines, 1967a) and at the 1967 Spring Joint Computer Conference (Gaines, 1967b). Ted Poppelbaum at the University of Illinois contacted me as he was an independent co-inventor of similar techniques for pattern recognition systems (Poppelbaum *et al.* , 1967). I visited him at Champaign, Urbana in 1967 and we became close friends, exchanging papers and research reports until he died in 1994.

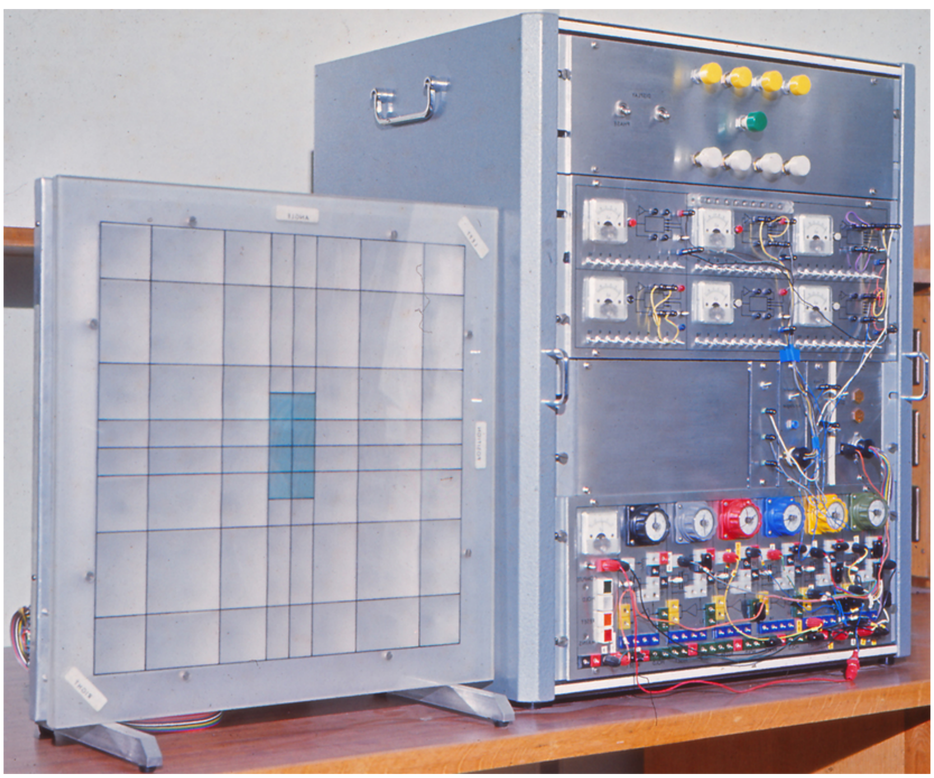

Figure 3: On right, stochastic analog computer built by Brian Gaines & Peter Joyce; on left, visual display of STelLA learning controller's trajectory to the specified region of state space—Standard Telecommunication Laboratories, 1965

I proved that a Rosenblatt *Perceptron* with digital weights did not converge because it could not implement steepest descent and went into limit cycles, but that a variant of the Novikoff convergence proof did apply to stochastic perceptrons built with stochastic integrators (Gaines, 1967c). I saw the asynchrony between random sequences as generating a requisite variety of behaviours that led to the *possibility* of convergence, and the existence of optimal trapping states as leading to its *necessity*, a result consistent with Ross Ashby's (1952) notion of inherent adaptivity in a system with many states of equilibrium.

In 1968 when Ladislav Kohout came to be my graduate student at the University of Essex he referred me to Rescher's (1963) proof of equivalence between probabilistic logic and the modal logic S5, which eventually led to reports and papers on a *calculus of possibility, probability and eventuality* (Gaines, 1975a) that encompassed fuzzy logic, and *possibilistic automata theory* (Gaines & Kohout, 1975a) based on the calculus.

## 6 Linguistic priming of controllers

The community of those researching machine learning, neural networks and artificial intelligence in the UK in the 1960s was fairly small and the members well-known to one another. I met with Igor Aleksander, Mike Brady, Jim Doran, Pat Hayes, Abe Mamdani, Donald Michie, Ted Newman, Gordon Pask, Pete Uttley, Yorick Wilks, and others, at various meetings within a cybernetics, artificial intelligence, machine learning, and pattern recognition ethos. I knew Abe as Igor's student working on the *SLAM* deterministic digital adaptive modules applied to pattern recognition (Aleksander & Mamdani, 1968), and Abe knew me as working on stochastic digital adaptive modules applied to learning machines and to modelling human adaptive control of unstable systems. We used the new IEE journal, *Electronics Letters*, as a vehicle for rapid publication as it guaranteed publication (or rejection) within 6 weeks, and I refereed some of Igor and Abe's paper and suspect they refereed some of mine.

Alexander Luria from the USSR was a friend of Oliver Zangwill's and a frequent visitor to the Experimental Psychology Department at Cambridge. I was fascinated by his research on the positive impact of verbal behaviour on performance of perceptual-motor skills (Luria, 1961) and built this into the experimental design for my studies of training human operators, investigating the trade-off between *priming* my subjects with helpful control strategies and non-verbal training techniques, and collecting their verbalizations in the very difficult control task I had set them.

My results demonstrated a strong effect of such priming and, when I attempted to show that the success of my training techniques had little or no dependence on human psychology but were systemic and would apply to any learning automaton capable of carrying out the task, I wanted to be able to prime my stochastic neural networks with the same verbal input as I had provided my subjects.

I did so by having the stochastic Perceptron *imagine* itself with the input specified, taking the action specified, and *rewarding* itself for so doing (Gaines, 1972b). This enabled me to replicate the positive effect of verbal instructions on my human subjects with identical phenomena in artificial adaptive controllers, demonstrating that the effects of coding, priming and training were all *cybernetic* phenomena in Wiener and Ashby's terms, a major thrust of my doctoral thesis which was very positivistic and behaviourist in keeping with the ethos of experimental psychology at that time.

## 7 The genesis of fuzzy control

Abe was at presentations I made on this research at IEE Control System Colloquia, had a copy of my thesis, and in 1971, when he completed his doctoral research and was appointed a lecturer at Queen Mary College, set his graduate student, Sedrak Assillian, the task of replicating the results using a realistic engineering situation, the control of a small steam engine, a task known to involve non-linearities and time-varying behaviour that was not amenable to linear modelling and optimal control approaches.

By that time I was Reader in the Department of Electrical Engineering Science, Essex University, Technical Director of two companies I had founded in 1968, one offering timeshared computer services, the other a minicomputer I had designed, and executive editor of the International Journal of Man-Machine Studies (IJMMS) that John Gedye and I founded in 1968. I was very busy and cannot recollect whether I even knew of the research or met Sedrak before Abe asked me to be his doctoral examiner in 1974.

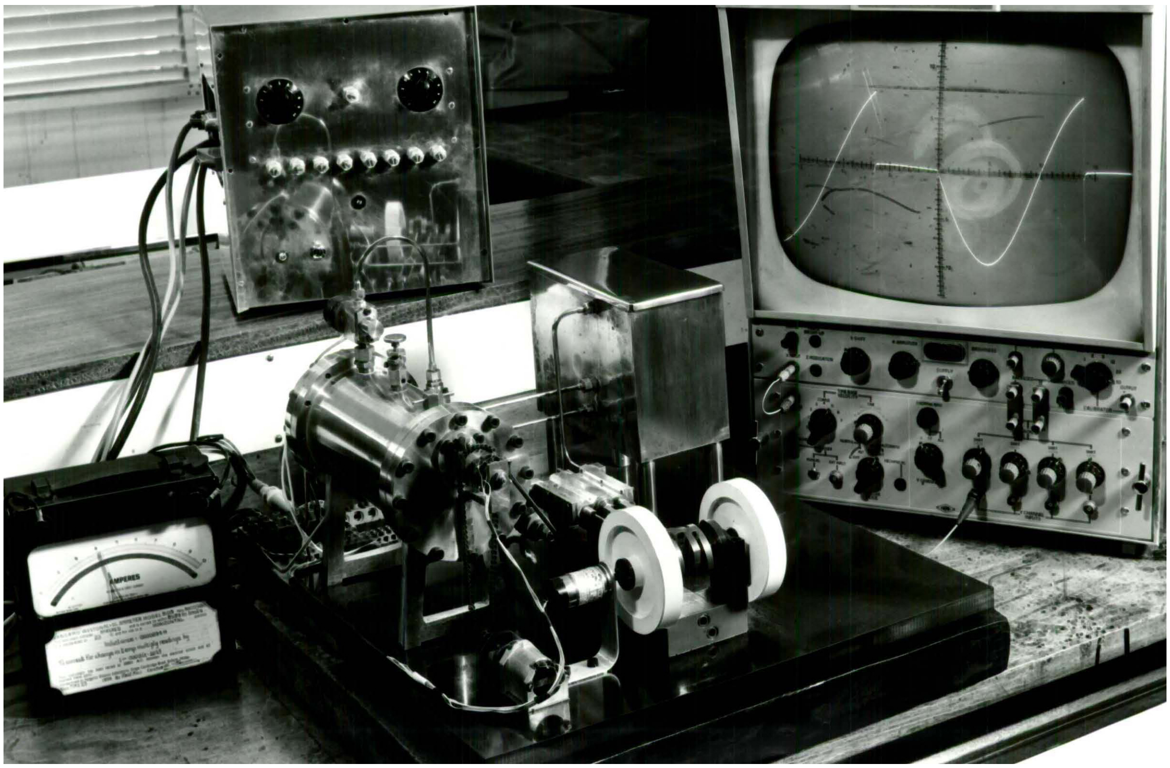


Figure 4: Abe Mamdani and Sedrak Assilian's steam engine, QMC, 1974

Sedrak's thesis (Assilian, 1974) is a model of scholarship. He considers the relative merits of Perceptron-like adaptive threshold logic elements (ATLE) and Bayesian learning elements as adaptive controllers, analyses the impact of different input and output encodings on their potential learning performance, and studies empirically both types of learning controller, finding little difference between them. For his first study he used human operators as targets for the learning controllers to emulate, and got poor results which he attributes to erratic control strategies that were difficult to emulate, possibly resulting from operator fatigue as the training took hours at a time over several days.

For the second study he used a digital controller as the role model and again got poor results which he attributes to the controller again being difficult to emulate but this time because it was using analog inputs and making finer distinctions than were available to the learning controller using quantized inputs. His third study was retrospective in that he used the "fuzzy logic controller" described in the second half of his thesis as a trainer to be emulated by his learning controllers and found that the ATLE could emulate it perfectly indicating that the results of his first two experiments were not defects of his learning algorithms.

The second half of the thesis investigated priming the learning system with a linguistic description of a suitable control policy. He notes that I translated linguistic hedges into precise inputs whereas Waterman translated them to a range of values, and proposes to use fuzzy logic to do so automatically using Lotfi's techniques for representing conditional statements in linguistic variables described his 1973 paper, *Outline of a new approach to the analysis of complex systems and decision processes* (Zadeh, 1973). He defined 8 linguistic variables (positive big, etc) for his two sets of inputs and outputs and represented them as fuzzy distributions over his quantized input and output variables. He developed heuristic linguistic rules based on his own experience for controlling the steam engine, tested them, identified where control was ineffective and adjusted the rules (but not the fuzzy distributions) for better control.

One can summarize Sedrak's achievement to be that of providing the human trainer with a means to communicate linguistically with a control system and to monitor the impact of specific instructions on the performance in order to improve it. He remarks that I had also found in my experiments that it was not possible to choose a good set of instructions in advance and that feedback as to the effect of different instructions was necessary to develop the best set.

We both had emulated the way in which human trainers of perceptual-motor skills in athletes provide verbal instruction and modify it in the light of its effect on the trainee. Where Sedrak's made a major advance was in using Lotfi's algorithms to provide a principled translation from linguistic rules to a control policy. It was also interesting that his tuning of the linguistic statements resulted in a control policy that was very effective and primed the controller so well that no further learning improved it.

## 8 Evolution of fuzzy control

My reaction to Sedrak's thesis was three-fold: first, to request a paper for IJMMS as I saw it as a major advance in linguistic interaction between people and computers; second, to see if fuzzy logic was necessary or whether I could replicate the results with stochastic elements; third; to investigate fuzzy logic further. Abe and Sedrak published *An experiment in linguistic synthesis with a fuzzy logic controller* (Mamdani & Assilian, 1975) in IJMMS in 1975 and it generated widespread interest.

I published *Stochastic and fuzzy logics* (Gaines, 1975b) in *Electronics Letters* in 1975 showing that one could substitute probability logic for fuzzy logic in interpreting Sedrak's linguistic specification and obtain the same control policy. Ladislav and I subsumed all forms of uncertainty in dynamical systems within the framework of *possibilistic automata*, unifying fuzzy and probabilistic sequential systems within an algebraic framework, and published *The logic of automa* (Gaines & Kohout, 1975a) in Gerge Klir's *International Journal of General Systems* in 1975.

Abe took these results in his stride citing them in his papers (Mamdani & Assilian, 1975; Mamdani, 1976) and noting that there were other ways of implementing linguistic heuristics that gave the same results as fuzzy logic. Neither of us saw the details of the implementation as important; it was the use of linguistic statements to communicate with a learning controller and lead it to develop an effective policy that was the breakthrough. In 2009 he still emphasized in his correspondence with Enric Trillas that "Fuzzy control should not be seen as an experimental proof of the correctness of fuzzy logic." (Mamdani & Trillas, 2012)

Abe had interested many in industrial control world-wide in the potential applications of linguistically specified heuristics in designing controllers for difficult plants that required human operators because conventional automatic control techniques were inapplicable. The instructions for the human operators in Peray and Waddell's (1972) book on *The Rotary Cement Kiln* were so similar in form to those Sedrak had used as to represent an independent confirmation of the industrial value of the approach.

Abe and I organized a series of workshops on *Discrete Systems and Fuzzy Reasoning* (Mamdani & Gaines, 1976), the first of which took place at Queen Mary College in January 1976 with a 300 page proceedings published by University of Essex. By 1982 the popular science press also found the potential applications of fuzzy reasoning intriguing. (Herman, 1982)

The original idea of fuzzy logic came from Lotfi Zadeh (left centre), professor at the University of California at Berkeley. He and other theorists have since developed some sophisticated mathematics to accompany the basic ideas. It is not yet clear, though, how the theory relates to other, more conventional, approaches to non-rigorous concepts and arguments. It is slightly worrying to the proponents of fuzzy logic that people working on "expert systems", for example for chemical analysis and medical diagnosis, have not needed to invoke Zadeh's work directly.

Abe Mamdani (left) lecturer in the Department of Electrical Engineering at Queen Mary College, London, worked with one of his students on a fuzzy-logic controller for a steam engine. Brian Gaines (far left), is one of the computer scientists whose ideas led to the use of fuzzy logic in process controllers. Gaines, formerly a professor of computer science at Essex University, is now a consultant based in London. □

Figure 5: From *Computing with a human Face, New Scientist,* May 1982

I had already published eight papers citing fuzzy sets in IJMMS prior to 1974. In the first volume in 1969 I had solicited one from Bill Kilmer and Warren McCulloch where Bill cites his paper on *Biological applications of the theory of fuzzy sets and fuzzy algorithms* (Kilmer, 1969). I asked Joe Goguen to submit his report on *Concept representation in natural and artificial languages: Axioms, extensions and applications for fuzzy sets* (Goguen, 1974) and published it in 1974. Gordon Pask (Pask *et al.* , 1973) in 1973 cites three papers by Lotfi on fuzzy algorithms in the context of Manna's non-deterministic algorithms and suggests that my work on *Axioms for adaptive behavior* (Gaines, 1972a) provides a bridge between the two; my paper cites Lotfi heavily for his work on adaptivity but not his later work on fuzzy sets—Gordon somehow made a link.

# 9 Visiting Lotfi at Berkely

In 1975 I was appointed Professor of Computer Systems at Essex University and Head of the Department of Electrical Engineering Science. I went to visit Ted Poppelbaum at Champaign, Illinois, and drove to Bloomington, Indiana, to present a paper by Ladislav and myself on *Possible automata* (Gaines & Kohout, 1975b) at the International Symposium on Multivalued Logic, where Ryszard Michalski suggested recasting the results within his *variable-valued logic* framework.

I went to Los Angles to visit Joe and Lotfi, gave a seminar at UCLA on my research on the identification of stochastic systems, and stayed at Joe's apartment. We went into UCLA's Moog Synthesizer Laboratory at 2am, and listened to the resulting 'music' while discussing whether my modelling process could be cast as a category-theoretic adjunction in a a similar way to his unification of the identification problems for linear systems and automata in his 1972 paper *Realization is universal* (Goguen, 1972).

I visited Lotfi at Berkely and gave a presentation on the QMC controller studies. I think this was the first time we had met but I knew his work on system theory and he knew mine on stochastic computing, and we quickly engaged in discussions of fuzzy sets, their relation to multi-valued logics, and their applications to heuristic control. His office shelves were stacked with large plastic laundry baskets containing reprints of papers from all over the world sent to him by those working on fuzzy sets and their applications, and he gave me permission to look through them, make notes and copy some.

I asked Lotfi for a paper for IJJMS and published *A fuzzy-algorithmic approach to the definition of complex or imprecise concepts* (Zadeh, 1976) in 1976. I published notes on my meeting with Lotfi, *Research notes on fuzzy reasoning* (Gaines, 1976b), in the 1976 workshop proceedings.

# 10 Fuzzy Logic Bibliography, Survey and Synthesis

When I returned to the UK I wrote to over 200 authors requesting copies of papers relating to fuzzy sets. Ladislav and I read them, classified them, developed an annotated bibliography, sent it to the authors for comment and published it as *The fuzzy decade: a bibliography of fuzzy systems and closely related topics* (Gaines & Kohout, 1977) comprising some 1,100 items.

In 1976 I published a paper, *Foundations of fuzzy reasoning* (Gaines, 1976a), setting fuzzy reasoning within the framework of the relevant literature on multi-valued logics, noting that fuzzification of logic left the definition of implication open, and suggesting a range of possible fuzzy implication functions. It was interesting as I reviewed the current literature in writing this paper to find a number of these now associated with my name and used in a wide range of applications.

I published my synthesis of probability and fuzzy logics as a *Fuzzy and probability uncertainty logics* (Gaines, 1978) in *Information and Control* in 1978, suggesting that there was a *standard uncertainty logic* (SUL) subsuming probability and fuzziness, each of which could be derived from it by additional axioms: *excluded middle* and *truth functionality*, respectively. However, Sedrak's linguistic controller algorithm could be derived directly within a SUL (Gaines, 1983) and did not need commitment to either additional axiom.

## 11 Monotype and SGSR

In 1978 I left Essex University, having been head-hunted by the UK National Enterprise Board to be Technical Director and Deputy Chief Executive of the Monotype Corporation, a nineteenth century British printing equipment company that was in financial trouble but had the possibility of recovery through new computer technology. Monotype had some 4,000 employees, 26 subsidiaries around the world and was on the verge of bankruptcy; the job required a major time commitment and continuous travel. In 1978 also George Klir invited me to become President of the Society for General Systems Research which provided some balance between my industry and academic lives.

While at Monotype I continued to edit IJMMS, and managed to fit in the banquet speech at the First North American Fuzzy Information Processing Group Workshop at Utah State University, in May 1982, between my industry trips in North America—later published as *Precise past—fuzzy future* (Gaines, 1983). After that trip I collapsed on return to England and was harangued by a doctor who told me to buy some medical textbooks and treat my body with the same care as I did my computers.

## 12 Moving to North America

My wife, Mildred Shaw, and I decided that it might be time to slow down and emigrated to Canada in 1982 with funding from the Canadian government for me to form a research company developing handwriting recognition tablets, and a Professorship in Computer Science at York University for Mildred.

In 1985 David Hill, by then at the University of Calgary, nominated me for the Killam Research Chair there, and we remained at UofC for 15 years until retiring out to the West coast of Vancouver Island at the end of 1999. At Calgary we became heavily involved in knowledge acquisition for knowledge-based systems research, collaborated with John Boose to launch a series of three annual knowledge acquisition workshops in Banff, Europe and South East Asia, and an associated journal and book series. By that time fuzzy reasoning had become well-established and figured in many of the KAW papers.

## 13 Reflections in Retirement

In the summer of 2004, after 36 years, I retired as editor of IJHCS (an updated title for IJMMS, also incorporating the KA journal) and stopped travelling. My research interests include: the role of knowledge in civilization, from the origins of *homo erectus* to the present day and beyond (Gaines, 2011); knowledge acquisition, representation and inference methods and tools (Gaines, 2009; Gaines & Shaw, 2012); the nature of human rationality in everyday reasoning and the sciences (Gaines, 2010); and in minimalist, defeasible substructural logics that are adequate to model much human reasoning, solve many AI problems, and could have been developed by Aristotle within the framework of syllogistics.

For me the most significant issue in understanding human knowledge processes over the millennia is how we coordinate our activities by communicating using concepts that are open and ill-defined, often with no common agreement on their meaning and usage. Even in close-knit scientific communities there can be commonly used concepts that are interpreted in very different ways (Shaw & Gaines, 1989).

Human civilization 'muddles through' (Fortun & Bernstein, 1998) very effectively in a way that seems foreign to logic and rationality. However, one can also view the phenomena as resulting from our intrinsic uncertainties about the world being accurately communicated through our use of appropriately vague terms in our languages. We convey not only the properties of the world but also our uncertainties about them; this is a rational process of the social brain that we need to model in our theories and associated *soft computing* tools.

# 14 Concluding Remarks

I will finish as I started by remarking on how privileged I have been to have friendships and intellectual dialogue with so many remarkable people. Abe and I had a lot of fun together and it was fascinating to see his ideas propagate into a control industry that had achieved so much based on linear systems theory but had a substantial residue of processes that were not tractable within that framework. I have often remarked that there is no such thing as 'nonlinear systems theory'—it is the heterogenous mess that is left when the linearization paradigm fails—Abe made a significant dent in that mess.

I see papers attempting to reconstruct the basis of those achievements and will echo what Abe has said—you need to go back to Sedrak's thesis for such reconstruction; it is readily available on the web from QMC (Assilian, 1974).

Lotfi and I continue to communicate although our paths no longer cross. It is a pleasure to have this opportunity to say how much I respect and admire all he has achieved from his seminal analyses of the notions of *state* and *adaptivity* to his development of computational algorithms for *computing with words* that have inspired so many theoretical innovations and industrial applications.

I have enjoyed re-reading much of the early literature and catching up with the recent literature in writing this article, and look forward to seeing the other contributions in this book. One privilege of having been involved in this community is the insights it provides into how new constellations of knowledge form in our society.

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