CPSC 351 — Tutorial Exercise #13
Additional Practice Problems

This problem will not be discussed during the tutorial, and solutions for this problem will not be made available. It can be used as a "practice" problem that can help you practice skills considered in the lecture presentation for Lectures #17–18, or in Tutorial Exercise #13.

In the preparatory material for Lecture #18, a language "All_{TM}" was proved to be undecidable. This was the language of Yes-instances of the following decision problem:

The "Accepting All" Problem

Instance: A Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$.

Question: Does M accept every input string? That is, is $L(M) = \Sigma^\star$?

So one could also say that the "Accepting All" Problem was proved to be undecidable.

Consider the following the following decision problem:

The "Same Language" Problem

Instance: A pair of of Turing machines, M_1 and M_2, with the same input alphabet.

Question: Does M_1 and M_2 have the same language? This is, is $L(M_1) = L(M_2)$?

1. To begin, describe a high-level mapping that can (probably) be used to show that if the "Accepting All" problem is undecidable then the "Same Language" problem is also undecidable too.

In order to continue, it is necessary to consider encodings of instances of the "Same Language" problem. Let $\Sigma_{\text{2TM}} = \Sigma_{\text{TM}} \cup \{\#\}$.

- A *pair* of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma_{\text{2TM}}^\star$ where $\alpha \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_1 and $\beta \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_2.
- Let $\text{Pair}_{\text{TM}} \subseteq \Sigma_{\text{2TM}}^\star$ be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

with the same input alphabet Σ.

- Now let

  $$\mathsf{E_{TM}} \subseteq \mathsf{Pair_{TM}} \subseteq \Sigma_{\mathsf{2TM}}^{\star}$$

  be the language including encodings of pairs of Turing machines M_1 and M_2, with the same input alphabet Σ, such that $L(M_1) = L(M_2)$.

2. Sketch a proof that the language $\mathsf{Pair_{TM}}$ is decidable.

3. Prove that $\mathsf{All_{TM}} \preceq_{\mathsf{M}} \mathsf{E_{TM}}$ — so that the language $\mathsf{E_{TM}}$ is undecidable.