

CPSC 351 — Tutorial Exercise #13

Many-One Reductions II

This question [s] is intended to give you practice in establishing **many-one reductions** between languages. It is of the difficulty, and length, that would be appropriate for a question on an **assignment** in CPSC 351.

Problem To Be Solved

1. Consider the following decision problem.

The Rejection Problem

Instance: A Turing machine

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

and an input string $\omega \in \Sigma^*$ for M .

Question: Does M **reject** M ?

Let us use the same alphabet Σ_{TM} and encoding for Turing machines and input strings as in Lecture #15, so that the decidable language $L_{\text{TM}+I} \subseteq \Sigma_{\text{TM}}^*$, introduced in that lecture, is the *language of instances* of this decision problem. Let $\text{Reject}_{\text{TM}} \subseteq L_{\text{TM}+I}$ be the *language of Yes-instances* of this decision problem.

Prove that the Rejection Problem is undecidable — that is, prove that the above language, $\text{Reject}_{\text{TM}}$, is undecidable.

A **hint** for this problem is available in a separate file — but you should spend at least a little bit of time trying to solve this problem, without looking at it, before you use this hint.