

CPSC 351 — Tutorial Exercise #11

Oracle Reductions

These questions are intended to give you practice in establishing **oracle reductions** between languages. They are of the difficulty, and length, that would be appropriate for questions on a **test** in CPSC 351.

Problems To Be Solved

1. Let Σ be an alphabet, let $L \subseteq \Sigma^*$, and consider the language

$$L \circ L = \{\omega_1 \cdot \omega_2 \mid \omega_1, \omega_2 \in L\} \subseteq \Sigma^*.$$

Prove that $L \circ L \preceq_O L$.

2. Let Σ be an alphabet. The **reversal**, ω^R , of a string $\omega \in \Sigma^*$, is the string obtained by reversing the order of the symbols in the string. That is, if

$$\omega = \alpha_1 \alpha_2 \dots \alpha_{n-1} \alpha_n$$

where $\alpha_1, \alpha_2, \dots, \alpha_{n-1}, \alpha_n \in \Sigma$ (so that n is the length of ω), then

$$\omega^R = \alpha_n \alpha_{n-1} \dots \alpha_2 \alpha_1.$$

- (a) Show that the function $f : \Sigma^* \rightarrow \Sigma^*$ such that $f(\omega) = \omega^R$, for every string $\omega \in \Sigma^*$, is a computable function.

Let $L \subseteq \Sigma^*$. Let L^R be the set of reversals of strings in L , that is,

$$L^R = \{\omega^R \mid \omega \in L\}.$$

- (b) Prove that $L^R \preceq_O L$.
- (c) Could the above result be used to prove that if L is **undecidable** then L^R is **undecidable** as well? If the answer is “no” then what reduction(s) could you give, to prove this, instead?