CPSC 351 — Tutorial Exercise #8
Nonregular Languages, Part Two

About This Exercise

The goal of this exercise is to help you to understand, and give you practice with, the use of closure properties to prove that languages are not regular.

Getting Started

This initial problem will probably not be discussed during the tutorial. Please discuss it during office hours with the instructor, if you can, if you have trouble solving them.

1. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let $L = \emptyset$. Please identify the ***error*** that has been made, in the following incorrect proof, as precisely as you can — assuming that you have already proved that the languages L_1 and L_2 are not regular. (This error is similar to one that students have made, when writing proofs that languages are not regular, in the past.)

   Claim: The above language, L, is not regular.

   Proof. Recall that the languages

   $$L_1 = \{\mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{Z} \text{ and } n \geq 0\}$$

   and

   $$L_2 = \{\mathtt{a}^n\mathtt{b}^m \mid n \in \mathbb{Z},\, m \in \mathbb{Z},\, n, m \geq 0 \text{ and } n \neq m\}$$

   are languages such that $L_1 \subseteq \Sigma^\star$, $L_2 \subseteq \Sigma^\star$, L_1 is not regular, and L_2 is not regular. Since $L_1 \cap L_2 = \emptyset = L$ it follows that the language $L = \emptyset$ is not regular, as well. $\square$

Problems To Be Discussed

2. Let $\Sigma = \{\mathsf{a}\}$ and recall, from the previous exercise, that the language

$$L_p = \{\mathsf{a}^n \mid n \text{ is a prime number}\} \subseteq \Sigma^\star$$

is *not* a regular language. Recall, as well, that a positive integer n is ***composite*** if $n \geq 2$ and there exist integers k and ℓ such that $2 \leq k, \ell \leq n-1$ and $k \times \ell = n$. Every positive integer n, such that $n \geq 2$, is either prime or composite — but not both. On the other hand, the integers 0 and 1 are neither prime nor composite.

Let $L_c \subseteq \Sigma^\star$ be the language

$$L_c = \{\mathsf{a}^n \mid n \text{ is composite}\}.$$

Use this information, and one or more of the ***closure properties*** for regular languages that have now been introduced in this course, to prove that L_c is *not* a regular language.