

Lecture #10: Nonregular Languages, Part Two

What To Do Before the Lecture

1. Complete the preparatory viewing or reading for Lecture #10 — nothing that each video will probably be understandable if you play it at double speed.
2. **Print** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to check that you understand how to use closure properties to prove that a given language is not regular.

Problem To Be Solved

1. Consider **closure properties** for regular languages.
 - (a) List as many closure properties for regular languages, that have been stated in this course at this point, as you can.

(b) Describe how these can be used to prove that languages are *not* regular.

2. Let $\Sigma = \{a, b\}$. We wish to prove that the language

$$L = \{\omega \in \Sigma^* \mid \text{the number of a's in } \omega \text{ is equal to the number of b's in } \omega\} \subseteq \Sigma^*$$

is not a regular language.

(a) State a reasonably similar language to this one that we already know is not regular.

(b) State, as precisely as you can, how this language is related to the above language L .

- (c) Using this information — and one or more closure properties (*without* using the Pumping Lemma) to prove that L is not regular.

3. Consider the following.

Claim: For every alphabet Σ and for languages $L_1, L_2 \subseteq \Sigma^*$, if L_1 is a regular language and L_2 is a regular language then their **intersection**, $L_1 \cap L_2$, is also a regular language.

(a) Use **De Morgan's Laws** to write $L_1 \cap L_2$ in another way (also expressing this as a function of L_1 to L_2 — but using the complements and unions of sets, instead of using set intersection).

(b) Use this to write a proof of this claim.

(c) Use this to write a *shorter* and *simpler* proof that the language

$$L = \{\omega \in \Sigma^* \mid \text{the number of a's in } \omega \text{ is equal to the number of b's in } \omega\} \subseteq \Sigma^*$$

is not a regular language.

