

Lecture #9: Nonregular Languages, Part One

What To Do Before the Lecture

1. Complete the preparatory viewing or reading for Lecture #9 — nothing that each video will probably be understandable if you play it at double speed.
2. **Print** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to check that you understand how to use the Pumping Lemma to prove that a given language is not regular.
3. The supplemental material also include another example of the use of the Pumping Lemma to prove that a language is not regular, and (for interest only) a proof of this technical result.

Problem To Be Solved

1. Using the Pumping Lemma, we wish to show, for $\Sigma = \{a, b\}$, that the language

$$L = \{a^{(n^2)} \mid n \in \mathbb{N}\} \subseteq \Sigma^*$$

is not regular.

- (a) What **proof technique** should be used, when the “Pumping Lemma” is to be applied to prove that a language is not regular?

(b) Why is it often a good idea to write the “Pumping Lemma” out, near the beginning of a proof where you are using it?

(c) What gets introduced (in the argument) right away? What, if anything, can be assumed about this? Why?

(d) The **second** thing that gets introduced, is a string s . What properties do you need to show that this string has? Why?

(e) List some reasonable choices for the “string s ” that you might choose, when you are trying use the “Pumping Lemma for Regular Languages” to prove that the above language, L , is not regular.

- (f) What else must be introduced, and what more do you need to do, to complete the proof?

- (g) List the points that must be made (or the things that must be done) to finish — explaining *why* the various things, that must be established, are true when you use the above string s .

- (h) Use the above to write a proof that the language L is not regular — that uses the above ideas, but that is organized and written in a way that make it as easy as possible for another reader to understand and believe it.

