Lecture #7: Regular Operations and Regular Expressions

What To Do Before the Lecture

1. Complete the preparatory viewing or reading for Lecture #7 — nothing that each video will probably be understandable if you play it at double speed.

2. ***Print*** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to check that you understand the regular operations, as well as the definitions of a regular language and the language of a regular expression. They should also help you to solve *simple* problems involving regular expressions — including designing a regular expression for a given language, deciding whether a string of symbols *is* a regular expression over an alphabet, and — when given a regular expression — deciding (and proving) whether a given string belongs to the language of this regular expression, and describing its language.

   The supplemental material also includes material that is for interest only. This includes proofs of claims, about relationships between regular operations and regular languages, as well as relationships between regular expressions and regular languages, that were stated in the preparatory material for this lecture.

Problems To Be Solved

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ —- so that this language does not include any of the symbols

$$\lambda, \emptyset, \Sigma, (,), \cup, \circ, ^\star$$

and let

$$\Sigma_{\text{regexp}} = \Sigma \cup \{\lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\}.$$

Consider the string

$$\omega = ((((\Sigma)^\star \circ \mathtt{a}) \circ (\Sigma)^\star) \circ (\mathtt{a} \circ (\Sigma)^\star)) \in \Sigma^\star_{\text{regexp}}.$$

1. Using the definition of a ***regular expression***, prove that ω is a regular expression over Σ.
   Solution:

Here is *another* way to write this.

2. Consider the string $\mu = \texttt{abaca} \in \Sigma^\star$. Use the definition of the language of a regular expression, and the solution for the first problem (as needed), to prove that μ is in the language of ω.

   Solution:

3. Describe (in clear and simple English) the language of the regular expression ω. Was the solution for the first problem helpful, as you tried to solve this one? If it was, then please say how.

The Language of the Regular Expression ω:

Was the Solution for the First Problem Helpful, Here? If so, How?

4. Once again, let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and let L be the set of all strings in $\Sigma^\star$ that include an even number of copies of the symbol "a" — that is, the language

$$L = \{\mu \in \Sigma^\star \mid \text{the number of copies of "a" in } \mu \text{ is divisible by } 2\}.$$

Design a regular expression, over Σ, whose language is L.

Strategy for the Discovery of a Regular Expression for a Given Language:

Application To This Example: