Lecture #3: DFA Design and Verification — Part One

What To Do Before the Lecture

1. Complete the preparatory viewing or reading for Lecture #3 — noting that each video will probably be understandable if you play it at double speed.
2. ***Print*** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to begin (but, not necessarily complete the process, for design of a deterministic finite automaton, that is presented.

   Note that a ***supplemental document*** includes another design example. This might be helpful if you are having trouble understanding the design process that you are to use.

Problems To Be Solved

1. Once again, let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let

   $$L = \{\omega \in \Sigma^\star \mid \text{either } \omega \text{ does not include an “a” or } \omega \text{ does not include a “b”}\}.$$

   Recall that, when designing a deterministic finite automaton for L, we considered a pair of sets:

   $$S_0 = L = \{\omega \in \Sigma^\star \mid \text{either } \omega \text{ does not include an “a” or } \omega \text{ does not include a “b”}\}$$

   and

   $$S_1 = \Sigma^\star \setminus L = \{\omega \in \Sigma^\star \mid \omega \text{ includes both an “a” and a “b”}\}.$$

   We were trying to design a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, whose language is L, such that $Q = \{q_0, q_1\}$ — and such that

   $$S_0 = \{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_0\} \qquad \text{and} \qquad S_1 = \{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_1\}.$$

   Unfortunately, this attempt was not successful — because we discovered that the transition out of state q_0 for the symbol “a” was not well-defined.

In order to consider a transition out of state q_0 for the symbol "b", we must consider the set

$$\{\omega \cdot \mathtt{b} \mid \omega \in S_0\}.$$

Let $\omega \in S_0$. Then three cases might arise.

(a) *Case:* ω does not include any a's *or* any b's.

(b) *Case:* ω includes at least one "a" but does not include any b's.

(c) *Case:* ω incudes at least one "b" but does not include any a's.

(a) *Case:* ω does not include any a's *or* any b's:

(b) *Case:* ω includes at least one "a" but does not include any b's:

(c) *Case:* ω includes at least one “b” but does not include any a’s:

Conclusion:

2. Once again, let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and consider the language

$$L = \{\omega \in \Sigma^\star \mid \omega \text{ ends with } \mathtt{abb}\}.$$

Let us try to use the design process, introduced in the preparatory material, to design a deterministic finite automaton (with alphabet Σ) whose language is this language, L.

What Must the DFA Remember?

Representation Using Subsets of $\Sigma^\star$

Initial Sanity Checks

Identifying Transitions

What Went Wrong: Which Transition was not Well-Defined? Why?