Lecture #2: Introduction to Deterministic Finite Automata

What To Do Before the Lecture

1. Look at the beginning of the material in the "Finite Automata and Regular Languages" part of the course web site — up to the beginning of the material for Lecture #1.

2. Complete the preparatory viewing or reading for Lecture #2 — noting that each video will probably be understandable if you play it at double speed.

3. ***Print*** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to make sure that you understand how deterministic finite automata process strings, and use proof techniques — that you learned about in CPSC 251 or MATH 251 — to prove things about these automata.

Problems To Be Solved

Consider a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that has alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and that can be represented as shown in Figure 1 on page 2. One can see from this picture that $Q = \{q_0, q_1, q_2\}$, q_0 is the start state (as the representation of M as a 5-tuple also shows) and that $F = \{q_0\}$.

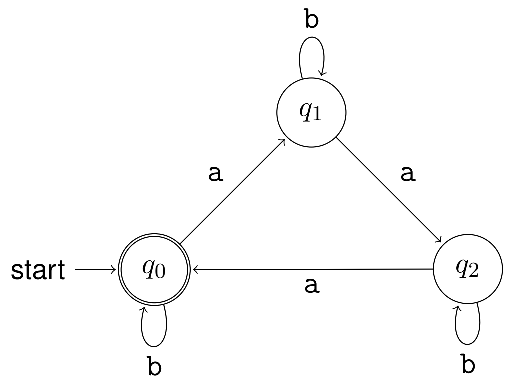

Figure 1: A Deterministic Finite Automaton

1. Consider the ***transition function*** $\delta : Q \times \Sigma \to Q$ for this deterministic finite automaton.

   (a) Complete this specification of this function.

   $\delta(q_0, \mathtt{a}) =$ $\delta(q_1, \mathtt{a}) =$ $\delta(q_2, \mathtt{a}) =$

   $\delta(q_0, \mathtt{b}) =$ $\delta(q_1, \mathtt{b}) =$ $\delta(q_2, \mathtt{b}) =$

   (b) Give a ***transition table*** for this deterministic finite automaton.

2. Consider the string $\omega = \texttt{abaabb}$.

(a) Complete the following table to compute $\delta^\star(q_0, \omega)$.

$$\delta^\star(q_0, \lambda) =$$

$$\delta^\star(q_0, \texttt{a}) =$$

$$\delta^\star(q_0, \texttt{ab}) =$$

(b) Complete the following derivation of $\delta^\star(q_0, \omega)$ (which uses the definition of the extended transition function directly).

$$\begin{aligned} \delta^\star(q_0, \omega) &= \delta^\star(q_0, \texttt{abaabb}) \\ &= \delta(\delta^\star(q_0, \texttt{abaab}), \texttt{b}) \\ &= \end{aligned}$$

3. Let $S_0, S_1, S_2 \subseteq \Sigma^\star$ be as follows.

   - $S_0 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 0 modulo $3\}$
   - $S_1 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 1 modulo $3\}$
   - $S_2 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 2 modulo $3\}$

   Consider the following.

   Claim 1. *Consider the above deterministic finite automaton M, and the above sets S_0, S_1, and S_2. The following properties are satisfied for every string $\omega \in \Sigma^\star$.*

   (a) $\delta^\star(q_0, \omega) = q_0$ *if and only if* $\omega \in S_0$.
   (b) $\delta^\star(q_0, \omega) = q_1$ *if and only if* $\omega \in S_1$.
   (c) $\delta^\star(q_0, \omega) = q_2$ *if and only if* $\omega \in S_2$.

   The goal, for the rest of this question, is to prove this claim.

   (a) Describe, as precisely as you can, a ***proof technique*** (which you studied in CPSC 251 or MATH 271) which could be used to prove this claim.

   (b) Ideally, you answered the above question by mentioned some form of ***mathematical induction*** — identifying both the form of mathematical induction to be used, and what you would be inducting on.

   To continue, state (as precisely as you can) ***what you need to prove*** in order to complete the ***basis*** for this proof.

(c) Given the outline of the ***inductive step***. As part of this, you should introduce a new parameter (possibly called k) that is to be used. You should also state *what can be assumed* and *what must be proved* in the inductive step.

(d) Using the above information, prove the claim.