

# Lecture #12: Discrete Probability for Computer Science III

## What To Do Before the Lecture

1. Watch the video for Lecture #12 — noting that it will probably be understandable if you play it at double speed. If you do not have time for this then look at the “Key Concepts” document that is found, immediately after the videos for this lecture, on the course web site, instead.
2. **Print** and read through the rest of this document and — if you have time — try to solve the problems!

## Problems To Be Solved

### Improving the Reliability of Randomized Algorithms

1. Recall, from Lecture #11, that a **Monte Carlo** algorithm is randomized algorithm for a decision problem that can, sometimes, return an incorrect answer — but that does so with small probability:
  - The algorithm halts, returning either `true` or `false`, when executed on any input (that is, any instance of the decision problem).
  - The algorithm only returns `true`, when executed on a given input, if this is the correct answer for that input.
  - If the algorithm is executed on an input where the answer that *should* be returned is `true`, then the probability that the algorithm *does* return `true` is at least  $\frac{1}{2}$ .

Describe a way to improve the reliability of this algorithm so that it returns the wrong answer with probability at most  $\varepsilon$ , for given real number  $\varepsilon$  such that  $0 < \varepsilon < 1$ .



2. Consider, instead, a randomized algorithm that can fail in *both* directions.

- The algorithm halts, returning either `true` or `false`, when executed on any input (that is, any instance of the decision problem).
- If the algorithm is executed on an input where the answer that should be returned is `true` then the probability that the algorithm returns `true` is at least  $\frac{3}{4}$  — so that the probability that the algorithm incorrectly returns `false` is at most  $\frac{1}{4}$ .
- On the other hand, if the algorithm is executed on an input where the answer that should be returned is `false` then the probability that the algorithm returns `false` is at least  $\frac{3}{4}$  — so that the probability that the algorithm incorrectly returns `true` is at most  $\frac{1}{4}$ .

Describe a way to improve the reliability of this algorithm so that it returns the wrong answer with probability at most  $\epsilon$ , for given real number  $\epsilon$  such that  $0 < \epsilon < 1$ .



## Classical Probability Distributions

2. Consider the examples from compute science — involving ***searching in arrays***, ***random walks***, and ***the analysis of algorithms for hashing***, that have been considered at the end of this course. Consider the ***classical probability distributions*** that have now been introduced, as well.

Try to identify parts of solutions for the problems from computer science that made use of one or more of the classical probability distributions that have now been introduced.

