CPSC 351 — Tutorial Exercise #8

Hint for the Final Problem in This Exercise

1. Consider the following decision problem.

   The Rejection Problem

   Instance: A Turing machine

   $$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

   and an input string $\omega \in \Sigma^\star$ for M.

   Question: Does M ***reject*** M?

   Let us use the same alphabet Σ_{TM} and encoding for Turing machines and input strings as in Lecture #8, so that the decidable language $L_{\text{TM+I}} \subseteq \Sigma_{\text{TM}}^\star$, introduced in that lectures, is the *language of instances* of this decision problem. Let $\text{Reject}_{\text{TM}} \subseteq L_{\text{TM+I}}$ be the *language of Yes-instances* of this decision problem.

   You were asked to prove that the Rejection Problem is undecidable — that is, prove that the above language, $\text{Reject}_{\text{TM}}$, is undecidable.

   Hint: Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ be a Turing machine. How could you make a *very simple* change, in order to produce another Turing machine

   $$\widehat{M} = (Q, \Sigma, \Gamma, \widehat{\delta}, q_0, q_{\text{accept}}, q_{\text{reject}})$$

   such that M rejects ω if and only if $\widehat{M}$ ***accepts*** ω, for *every* string $\omega \in \Sigma^\star$?

   Some of the changes that have been used, in other examples, are approximately as simple as the change that is needed here. Others are *more complicated* than the change that is needed here.