

More Turing Machine Variants and the Church-Turing Thesis

Supplement for Preparatory Viewing

Example of a Multi-Tape Turing Machine

Let $\Sigma = \{a, b\}$ and let

$$L = \{a^n b^n \mid n \in \mathbb{N}\}.$$

One Way To Decide Whether $\omega \in L$ — Using Two Tapes:

1. **Accept** if $\omega = \lambda$. Otherwise sweep right, copying initial copies of “a” onto the second type (marking the first so it can be found later) — **rejecting** if ω starts with “b”.
2. **Reject** if the next symbol seen, on the first tape, is “ \sqcup ”. Otherwise sweep **right** over copies of “b” on the first tape while matching with copies of “a” seen while sweeping back to the **left** on the second tape — **rejecting** if “a” is ever seen again on the first tape.
3. **Accept** if the leftmost copy of “a” is detected on the second tape, and matched with the rightmost “b” on the first tape (so that “ \sqcup ” is visible on the first tape after moving right) —and **reject** if one of these things happens before the other.

This can be implemented using a 2-tape Turing machine

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

where

$$Q = \{q_0, q_1, q_2, q_{\text{accept}}, q_{\text{reject}}\},$$

$$\Gamma = \{a, b, \dot{a}, \sqcup\},$$

and transitions are as shown in Figure 1 on page 2. In this figure, the accepting state is shown as “ q_A ” instead of “ q_{accept} ”, and transitions to the rejecting state (and the rejecting state, itself), are not shown — in order to keep the picture as simple as possible. Each missing transition should move to the rejecting state without changing the symbols visible on tapes or moving tape heads.

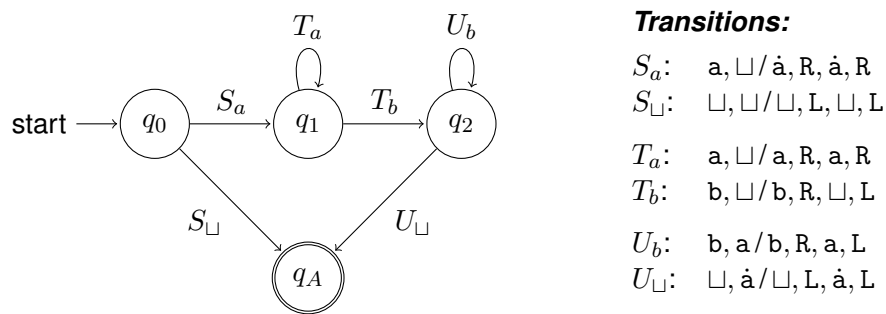
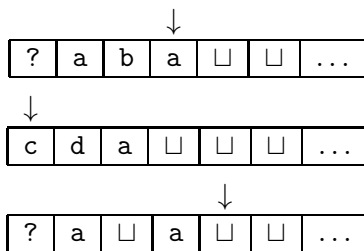


Figure 1: Multi-Tape Turing Machine Deciding the Language L

Simulation of a Multi-Tape Turing Machine

The simulation of a k -tape Turing machine, by a standard (single-tape) Turing machine, uses a greatly expanded tape alphabet — which includes symbols allowing a part of the tape to be thought of as including $2k$ **tracks**, allowing the positions of k tape heads, and contents of cells in k tapes, to be represented with one tape instead. For example, a configuration of a 3-tape Turing machine, whose tapes look like this,



would be representing with a single tape that looks like this:

