Lecture #3: Nondeterministic Finite Automata

What To Do Before the Lecture

1. Watch the videos for Lecture #3 —noting that they will probably be understandable if you play them at double speed. If you do not have time for this then look at the "Key Concepts" document that is found, immediately after the videos for this lecture, on the course web site, instead.

2. ***Print*** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to check that you understand how nondeterministic finite automata process strings, and that you are familiar with a process to use a nondeterministic finite automaton to produce a deterministic finite automaton with the same language.

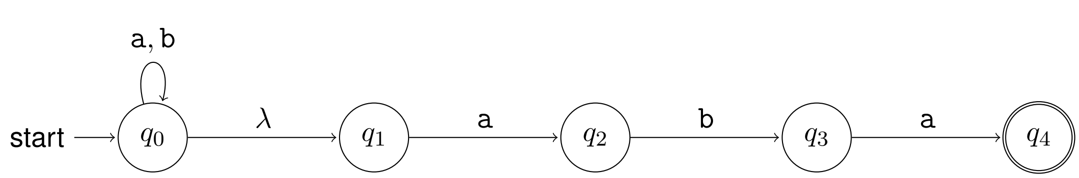
   The supplemental material also includes material that is for interest only. This includes additional information that you would need to write a program that decides membership in the language of a given nondeterministic finite automaton, along with proofs of claims that are stated in lecture material.

A Language and a Nondeterministic Finite Automaton

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let $L \subseteq \Sigma^\star$ be the following language:

$$L = \{w \in \Sigma^\star \mid \omega \text{ ends with } \mathtt{aba}\}.$$

Consider the following ***nondeterministic*** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ with the above alphabet Σ and the following transition diagram.



Problems To Be Solved

1. State the ***type*** of the transition function, δ, that is included as a part of this nondeterministic finite automaton.

   Solution:

2. Give a ***transition table*** for this transition function, δ.

   Solution:

3. Compute $Cl_{\lambda}(q)$ for every state, q, in this nondeterministic finite automaton.

   Solution:

4. Use the properties of the extended transition function, given in the lecture notes, to compute $\delta^\star(q_0, \texttt{aaba})$.

   Solution:

5. Use set-theoretic notation (if needed) to describe each of these sets, as precisely you can.

   (a) The set, S_0, of strings in $\Sigma^\star$ whose processing can end in state q_0, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_0 \in \delta^\star(q_0, \omega)\}$$

   Solution:

   (b) The set, S_1, of strings in $\Sigma^\star$ whose processing can end in state q_1, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_1 \in \delta^\star(q_0, \omega)\}$$

   Solution:

   (c) The set, S_2, of strings in $\Sigma^\star$ whose processing can end in state q_2, that is, the set

   $$\{\omega \in \Sigma^\star \mid q_2 \in \delta^\star(q_0, \omega)\}$$

   Solution:

(d) The set, S_3, of strings in $\Sigma^\star$ whose processing can end in state q_3, that is, the set

$$\{\omega \in \Sigma^\star \mid q_3 \in \delta^\star(q_0, \omega)\}$$

Solution:

(e) The set, S_4, of strings in $\Sigma^\star$ whose processing can end in state q_4, that is, the set

$$\{\omega \in \Sigma^\star \mid q_4 \in \delta^\star(q_0, \omega)\}$$

Solution:

6. Describe a ***proof technique***, that you learned about in a prerequisite for this course, that could be used to *prove* that S_0 is the set you claimed, when answering part (a) of the previous question.

   Solution:

   Simpler arguments could be given to prove the correctness of your claims about S_1, S_2, S_3, and S_4 after that. Why?

   Answer:

7. Assuming that your answers for the above questions are correct, explain why it now follows that the language of this nondeterministic finite automaton is the set of strings in $\Sigma^\star$ that end with `aba`.

   Solution:

8. Apply a process, described in the lecture material, to use the given nondeterministic finite automaton to produce a ***deterministic finite automaton*** with the same language.

   Solution: $\mathit{Cl}_\lambda(q)$ has already been computed for each state q in the given nondeterministic deterministic finite automaton, above. These will be used repeatedly when solving this problem.

   Initialization

   Our DFA, So Far:

   First Execution of the Body of the Main Loop

   Selecting a State whose Transitions should be Identified

   Computation of the Transition for the Symbol "`a`*"*

Computation of the Transition for the Symbol "`b`"

Our DFA, So Far

Reflections — What Have We Done? Which Strings Can Now Be Processed?

Second Execution of the Body of the Main Loop

Later Execution(s) of the Body of the Main Loop

Choosing the Accepting States

The DFA That Has Been Produced