Lecture #3: Nondeterministic Finite Automata
Completion of Example

Overview

Material for this lecture included a process that could be used a deterministic finite automaton

$$\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\delta}, \widehat{q}_0, \widehat{F})$$

with the same language as that of a given deterministic finite automaton

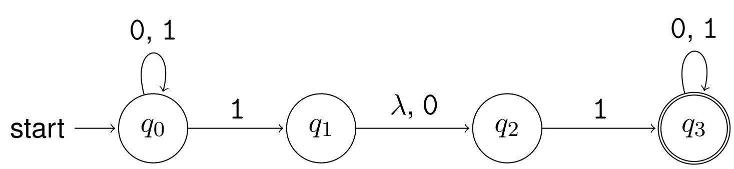
$$M = (Q, \Sigma, \Gamma, \delta, q_0, F).$$

To simplify the presentation, let us assume that

$$\Sigma = \{\sigma_0, \sigma_1, \dots, \sigma_{m-1}\}$$

where $m = |\Sigma|$. This process, that was described in the lecture material, is summarized in Figure 1 on page 2.

This document proves details for an application of this process to design a deterministic finite automaton, $\widehat{M}$, with the same language as that of the following nondeterministic finite automaton M:



The automata M and $\widehat{M}$ have the same alphabet, $\Sigma = \{0, 1\}$. Let us order the elements of Σ by setting σ_0 to be 0 and by setting σ_1 to be 1. Then, when the algorithm in Figure 1 is applied to compute the transitions out of a given state of $\widehat{M}$, the transition for 0 will be considered before the transition for 1.

1. Set $\widehat{q}_0$ to be a state such that $\varphi(\widehat{q}_0) = Cl_\lambda(q_0)$ — setting $\widehat{\delta}(\widehat{q}_0, \sigma)$ to be undefined, for every symbol $\sigma \in \Sigma$.
2. `integer i` := 1
3. $\widehat{R} := \{\widehat{q}_0\}$
4. $\widehat{Q} := \emptyset$
5. `while` $(\widehat{R} \neq \emptyset)$ `{`
6.   Choose a state $\widehat{q}$ from $\widehat{R}$; let V be the set of states, in the given NFA, such that $\varphi(\widehat{q}\,) = V$
7.   `integer j` := 0
8.   `while` $(\mathtt{j} < |\Sigma|)$ `{`
9. $W := \displaystyle\bigcup_{r \in V} \left(\bigcup_{s \in \delta(r, \sigma_{\mathtt{j}})} Cl_\lambda(s) \right)$
10.   Set $\widetilde{q}$ to be a (possibly new) state such that $\varphi(\widetilde{q}\,) = W$ — reusing an existing state, if one already exists; $\widehat{\delta}(\widehat{q}, \sigma_{\mathtt{j}})$ will be set to be $\widetilde{q}$
11.   `if` ($\widetilde{q}$ is a new state) `{`   // Update `i` and $\widehat{R}$
12. $\widehat{q}_{\mathtt{i}} := \widetilde{q}$
13. $\mathtt{i} := \mathtt{i} + 1$
14. $\widehat{R} := \widehat{R} \cup \{\widetilde{q}\}$

    `}`

15. $\mathtt{j} := \mathtt{j} + 1$

  `}`

16. $\widehat{R} := \widehat{R} \setminus \{\widehat{q}\}$
17. $\widehat{Q} := \widehat{Q} \cup \{\widehat{q}\}$

`}`

Figure 1: Construction to Compute the Set $\widehat{Q}$ of States and Transition Function $\widehat{\delta} : \widehat{Q} \times \Sigma \to \widehat{Q}$

Computation of λ-Closures

Since they will be used repeatedly (when the algorithm is applied) it is helpful to compute the λ-closures of states in the nondeterministic finite automaton M.

- Since there are no λ-transitions out of state q_0, $Cl_\lambda(q_0) = \{q_0\}$.

- Since there is a λ-transition out of state q_1, t the state q_2, but there are no other λ-transitions out of either q_1 or q_2, $Cl_\lambda(q_1) = \{q_1, q_2\}$.
- Since there are no λ-transitions out of either q_2 or q_3, $Cl_\lambda(q_2) = \{q_2\}$ and $Cl_\lambda(q_3) = \{q_3\}$.

Application of the Algorithm to This Example

Consider the application of the algorithm shown in Figure 1 to the nondeterministic finite automaton M that is given above.

Initialization

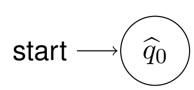
When the steps at lines 1–4 of the algorithm are executed, $\widehat{q}_0$ is set to be a state in the deterministic finite automaton, $\widehat{M}$ that is being designed, such that

$$\varphi(\widehat{q}_0) = Cl_\lambda(q_0) = \{q_0\}.$$

The integer variable `i` is initialized to have value 1.

The sets $\widehat{R}$ and $\widehat{Q}$ are initialized so that $\widehat{R} = \{\widehat{q}_0\}$ and $\widehat{Q} = \emptyset$.

The partial DFA that has been designed, so far, is as follows.



Determining Transitions out of $\widehat{q}_0$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the first time it is checked, and the body of the `while` loop (at lines 6–17) is executed at least once.

Now, since $\widehat{R}$ only includes one state — $\widehat{q}_0$ — the first time the step at line 6 is reached and executed, it is necessary to set $\widehat{q}$ to be $\widehat{q}_0$ at this point. In this case $V = \varphi(\widehat{q}_0) = Cl_\lambda(q_0) = \{q_0\}$.

The integer variable `j` is initialized to be 0 and the inner `while` loop (at lines 8–15) is reached and executed for the first time. One can see, by an examination of the code, that every execution of this loop will include two executions of its loop body. At the beginning of the first of these executions $\mathtt{j} = 0$, so that $\sigma_{\mathtt{j}} = \sigma_0 = 0$, and the steps at lines 9 and 10 will be used to identify $\widehat{\delta}(\widehat{q}, 0)$. At the beginning of the second of these executions $\mathtt{j} = 1$, so that $\sigma_{\mathtt{j}} = \sigma_1 = 1$, and the steps at lines 9 and 10 will be used to identify $\widehat{\delta}(\widehat{q}, 1)$.

Determining $\widehat{\delta}(\widehat{q}_0, \texttt{0})$

Since $V = \{q_0\}$ and $\sigma_{\texttt{j}} = \sigma_0 = \texttt{0}$, the first execution of the step at line 9 computes W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0\}} \left(\bigcup_{s \in \delta(r, \texttt{0})} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, \texttt{0})} \mathit{Cl}_\lambda(s) \\ &= \mathit{Cl}_\lambda(q_0) \\ &= \{q_0\} = \varphi(\widehat{q}_0). \end{aligned}$$

Thus $\widehat{\delta}(\widehat{q}_0, \texttt{0}) = \widehat{q}_0$. Since $\widehat{q}_0$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Determining $\widehat{\delta}(\widehat{q}_0, \texttt{1})$

Since $V = \{q_0\}$ and $\sigma_{\texttt{j}} = \sigma_1 = \texttt{1}$ at the beginning of the second execution of the inner loop body, this includes an execution of the step at line 9 computing W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0\}} \left(\bigcup_{s \in \delta(r, \texttt{1})} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, \texttt{1})} \mathit{Cl}_\lambda(s) \\ &= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1) \\ &= \{q_0\} \cup \{q_1, q_2\} \\ &= \{q_0, q_1, q_2\}. \end{aligned}$$

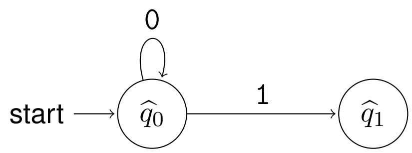
Since W is not equal to $\varphi(r)$ for any state $r \in \widehat{Q} \cup \widehat{R}$ a new state, "$\widetilde{q}$", is introduced such that $\varphi(\widetilde{q}) = W$, this state is given name $\widehat{q}_1$, and the value of $\texttt{i}$ is incremented, to have value 2, when the steps 11–14 are executed. Furthermore, $\widehat{q}_1$ is included in $\widehat{R}$ (when the step at line 14 is executed), so that $\widehat{R} = \{\widehat{q}_0, \widehat{q}_1\}$ at the end of this execution of the inner loop body.

Completing the First Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the first time $\widehat{q}_0$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0\} \quad \text{and} \quad \widehat{R} = \{\widehat{q}_1\}$$

at the end of the of the first execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Determining Transitions out of $\widehat{q}_1$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the second time it is checked, and the body of the `while` loop (at lines 6–17) is executed for a second time.

Now, since $\widehat{R}$ only includes one state — $\widehat{q}_1$ — when the step at line 6 is reached and executed, it is necessary to set $\widehat{q}$ to be $\widehat{q}_1$ at this point. In this case $V = \varphi(\widehat{q}_1) = \{q_0, q_1, q_2\}$.

The step at line 7 and the inner loop at lines 8–15 are used next to compute $\widehat{\delta}(\widehat{q}_1, \sigma)$ for $\sigma = 0$ and for $\sigma = 1$.

Determining $\widehat{\delta}(\widehat{q}_1, \mathtt{0})$

Since $V = \{q_0, q_1, q_2\}$ and $\sigma_{\mathtt{j}} = \sigma_0 = \mathtt{0}$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_1, q_2\}} \left(\bigcup_{s \in \delta(r, \mathtt{0})} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0, \mathtt{0})} \mathit{Cl}_\lambda(s) \;\cup \bigcup_{s \in \delta(q_1, \mathtt{0})} \mathit{Cl}_\lambda(s) \;\cup \bigcup_{s \in \delta(q_2, \mathtt{0})} \mathit{Cl}_\lambda(s) \\
&= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_2) \cup \emptyset \\
&= \{q_0\} \cup \{q_2\} \cup \emptyset \\
&= \{q_0, q_2\}.
\end{aligned}$$

Since W is not equal to $\varphi(r)$ for any state $r \in \widehat{Q} \cup \widehat{R}$ a new state, "$\widetilde{q}$", is introduced such that $\varphi(\widetilde{q}) = W$, this state is given name $\widehat{q}_2$, and the value of `i` is incremented, to have value 3, when the steps 11–14 are executed. Furthermore, $\widehat{q}_2$ is included in $\widehat{R}$ (when the step at line 14 is executed), so that $\widehat{R} = \{\widehat{q}_1, \widehat{q}_2\}$ at the end of this execution of the inner loop body.

Determining $\widehat{\delta}(\widehat{q}_1, 1)$

Since $V = \{q_0, q_1, q_2\}$ and $\sigma_{\mathtt{j}} = \sigma_1 = 1$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0, q_1, q_2\}} \left(\bigcup_{s \in \delta(r, 1)} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, 1)} \mathit{Cl}_\lambda(s) \ \cup \bigcup_{s \in \delta(q_1, 1)} \mathit{Cl}_\lambda(s) \ \cup \bigcup_{s \in \delta(q_2, 1)} \mathit{Cl}_\lambda(s) \\ &= (\mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1)) \cup \emptyset \cup \mathit{Cl}_\lambda(q_3) \\ &= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1) \cup \mathit{Cl}_\lambda(q_3) \\ &= \{q_0\} \cup \{q_1, q_2\} \cup \{q_3\} \\ &= \{q_0, q_1, q_2, q_3\}. \end{aligned}$$

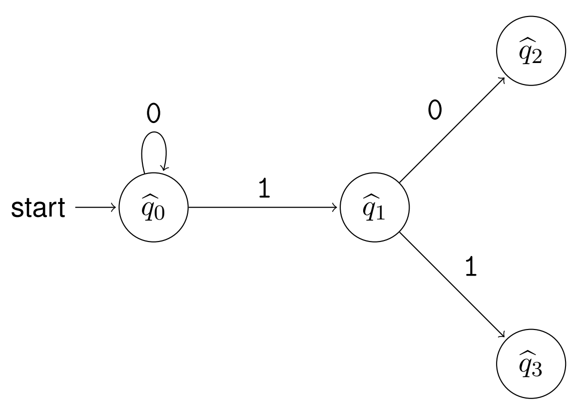
Since W is not equal to $\varphi(r)$ for any state $r \in \widehat{Q} \cup \widehat{R}$ a new state, "$\widetilde{q}$", is introduced such that $\varphi(\widetilde{q}) = W$, this state is given name $\widehat{q}_3$, and the value of $\mathtt{i}$ is incremented, to have value 4, when the steps 11–14 are executed. Furthermore, $\widehat{q}_3$ is included in $\widehat{R}$ (when the step at line 14 is executed), so that $\widehat{R} = \{\widehat{q}_1, \widehat{q}_2, \widehat{q}_3\}$ at the end of this execution of the inner loop body.

Completing the Second Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the second time $\widehat{q}_1$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0, \widehat{q}_1\} \quad \text{and} \quad \widehat{R} = \{\widehat{q}_2, \widehat{q}_3\}$$

at the end of the of the second execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Determining Transitions out of $\widehat{q}_2$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the third time it is checked, and the body of the `while` loop (at lines 6–17) is executed for a third time.

For the first time, $\widehat{R}$ includes more than one state — $\widehat{q}_2$ and $\widehat{q}_3$ — when the step at line 6 is reached and executed. It is possible to set $\widehat{q}$ to be either $\widehat{q}_2$ or $\widehat{q}_3$ at this point.

Let us set $\widehat{q}$ to be $\widehat{q}_\ell$ for the *smallest* integer ℓ such that $q_\ell \in \widehat{R}$. In particular, let us now set $\widehat{q}$ to be $\widehat{q}_2$.

In this case $V = \varphi(\widehat{q}_2) = \{q_0, q_2\}$.

The step at line 7 and the inner loop at lines 8–15 are used next to compute $\widehat{\delta}(\widehat{q}_2, \sigma)$ for $\sigma = 0$ and for $\sigma = 1$.

Determining $\widehat{\delta}(\widehat{q}_2, 0)$

Since $V = \{q_0, q_2\}$ and $\sigma_{\mathtt{j}} = \sigma_0 = 0$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_2\}} \left(\bigcup_{s \in \delta(r, 0)} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0, 0)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_2, 0)} \mathit{Cl}_\lambda(s) \\
&= \mathit{Cl}_\lambda(q_0) \cup \emptyset \\
&= \{q_0\} \cup \emptyset \\
&= \{q_0\} = \varphi(\widehat{q}_0).
\end{aligned}$$

Thus $\widehat{\delta}(\widehat{q}_2, 0) = \widehat{q}_0$. Since $\widehat{q}_0$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Determining $\widehat{\delta}(\widehat{q}_2, 1)$

Since $V = \{q_0, q_2\}$ and $\sigma_{\mathtt{j}} = \sigma_1 = 1$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0, q_2\}} \left(\bigcup_{s \in \delta(r, 1)} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, 1)} \mathit{Cl}_\lambda(s) \quad \cup \quad \bigcup_{s \in \delta(q_2, 1)} \mathit{Cl}_\lambda(s) \\ &= (\mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1)) \cup \mathit{Cl}_\lambda(q_3) \\ &= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1) \cup \mathit{Cl}_\lambda(q_3) \\ &= \{q_0\} \cup \{q_1, q_2\} \cup \{q_3\} \\ &= \{q_0, q_1, q_2, q_3\} = \varphi(\widehat{q}_3). \end{aligned}$$

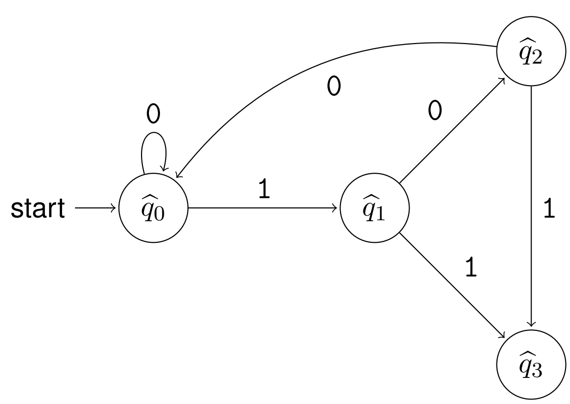
Thus $\widehat{\delta}(\widehat{q}_2, 1) = \widehat{q}_3$. Since $\widehat{q}_3$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Completing the Third Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the third time $\widehat{q}_2$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0, \widehat{q}_1, \widehat{q}_2\} \quad \text{and} \quad \widehat{R} = \{\widehat{q}_3\}$$

at the end of the of the third execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Determining Transitions out of $\widehat{q}_3$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the fourth time it is checked, and the body of the `while` loop (at lines 6–17) is executed for a fourth time.

Now, since $\widehat{R}$ only includes one state — $\widehat{q}_3$ — when the step at line 6 is reached and executed, it is necessary to set $\widehat{q}$ to be $\widehat{q}_3$ at this point. In this case $V = \varphi(\widehat{q}_3) = \{q_0, q_1, q_2, q_3\}$.

The step at line 7 and the inner loop at lines 8–15 are used next to compute $\widehat{\delta}(\widehat{q}_3, \sigma)$ for $\sigma = 0$ and for $\sigma = 1$.

Determining $\widehat{\delta}(\widehat{q}_3, \mathtt{0})$

Since $V = \{q_0, q_1, q_2, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_0 = \mathtt{0}$, the next execution of the step at line 9 computes W as follows:

$$
\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_1, q_2, q_2, q_3\}} \left(\bigcup_{s \in \delta(r, \mathtt{0})} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0, \mathtt{0})} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_1, \mathtt{0})} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_2, \mathtt{0})} \mathit{Cl}_\lambda(s) \cup \bigcup_{s \in \delta(q_3, \mathtt{0})} \mathit{Cl}_\lambda(s) \\
&= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_2) \cup \emptyset \cup \mathit{Cl}_\lambda(q_3) \\
&= \{q_0\} \cup \{q_2\} \cup \emptyset \cup \{q_3\} \\
&= \{q_0, q_2, q_3\}.
\end{aligned}
$$

Since W is not equal to $\varphi(r)$ for any state $r \in \widehat{Q} \cup \widehat{R}$ a new state, "$\widetilde{q}$", is introduced such that $\varphi(\widetilde{q}) = W$, this state is given name $\widehat{q}_4$, and the value of `i` is incremented, to have value 5, when the steps 11–14 are executed. Furthermore, $\widehat{q}_4$ is included in $\widehat{R}$ (when the step at line 14 is executed), so that $\widehat{R} = \{\widehat{q}_3, \widehat{q}_4\}$ at the end of this execution of the inner loop body.

Determining $\widehat{\delta}(\widehat{q}_3, 1)$

Since $V = \{q_0, q_1, q_2, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_1 = 1$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_1, q_2, q_3\}} \left(\bigcup_{s \in \delta(r,1)} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0,1)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_1,1)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_2,1)} \mathit{Cl}_\lambda(s) \cup \bigcup_{s \in \delta(q_3,1)} \mathit{Cl}_\lambda(s) \\
&= (\mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1)) \cup \emptyset \cup \mathit{Cl}_\lambda(q_3) \cup \mathit{Cl}_\lambda(q_3) \\
&= (\{q_0\} \cup \{q_1, q_2\}) \cup \emptyset \cup \{q_3\} \cup \{q_3\} \\
&= \{q_0, q_1, q_2, q_3\} = \varphi(\widehat{q}_3).
\end{aligned}$$

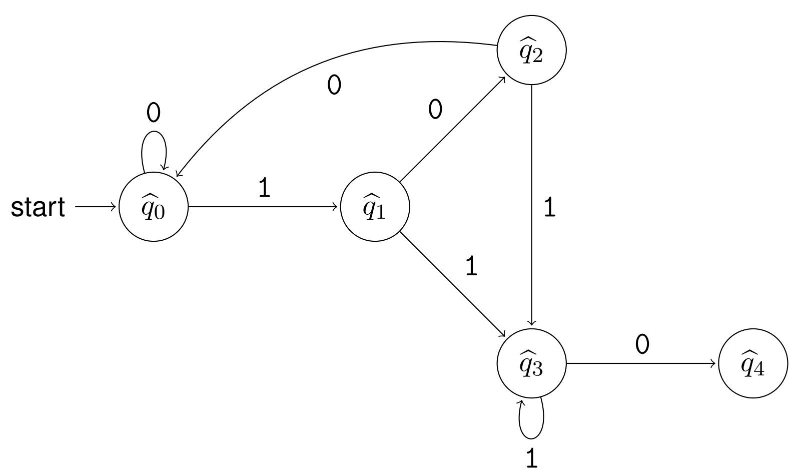
Thus $\widehat{\delta}(\widehat{q}_3, 1) = \widehat{q}_3$. Since $\widehat{q}_3$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Completing the Fourth Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the fourth time $\widehat{q}_3$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0, \widehat{q}_1, \widehat{q}_2, \widehat{q}_3\} \quad \text{and} \quad \widehat{R} = \{\widehat{q}_4\}$$

at the end of the of the fourth execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Determining Transitions out of $\widehat{q}_4$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the fifth time it is checked, and the body of the `while` loop (at lines 6–17) is executed for a fifth time.

Now, since $\widehat{R}$ only includes one state — $\widehat{q}_4$ — when the step at line 6 is reached and executed, it is necessary to set $\widehat{q}$ to be $\widehat{q}_4$ at this point. In this case $V = \varphi(\widehat{q}_4) = \{q_0, q_2, q_3\}$.

The step at line 7 and the inner loop at lines 8–15 are used next to compute $\widehat{\delta}(\widehat{q}_4, \sigma)$ for $\sigma = 0$ and for $\sigma = 1$.

Determining $\widehat{\delta}(\widehat{q}_4, \mathtt{0})$

Since $V = \{q_0, q_2, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_0 = \mathtt{0}$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_2, q_3\}} \left(\bigcup_{s \in \delta(r, \mathtt{0})} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0, \mathtt{0})} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_2, \mathtt{0})} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_3, \mathtt{0})} \mathit{Cl}_\lambda(s) \\
&= \mathit{Cl}_\lambda(q_0) \cup \emptyset \cup \mathit{Cl}_\lambda(q_3) \\
&= \{q_0\} \cup \emptyset \cup \{q_3\} \\
&= \{q_0, q_3\}.
\end{aligned}$$

Since W is not equal to $\varphi(r)$ for any state $r \in \widehat{Q} \cup \widehat{R}$ a new state, "$\widetilde{q}$", is introduced such that $\varphi(\widetilde{q}) = W$, this state is given name $\widehat{q}_5$, and the value of `i` is incremented, to have value 6, when the steps 11–14 are executed. Furthermore, $\widehat{q}_5$ is included in $\widehat{R}$ (when the step at line 14 is executed), so that $\widehat{R} = \{\widehat{q}_4, \widehat{q}_5\}$ at the end of this execution of the inner loop body.

Determining $\widehat{\delta}(\widehat{q}_4, 1)$

Since $V = \{q_0, q_2, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_1 = 1$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned}
W &= \bigcup_{r \in \{q_0, q_2, q_3\}} \left(\bigcup_{s \in \delta(r,1)} \mathit{Cl}_\lambda(s) \right) \\
&= \bigcup_{s \in \delta(q_0,1)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_2,1)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_3,1)} \mathit{Cl}_\lambda(s) \\
&= (\mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1)) \cup \mathit{Cl}_\lambda(q_3) \cup \mathit{Cl}_\lambda(q_3) \\
&= (\{q_0\} \cup \{q_1, q_2\}) \cup \{q_3\} \cup \{q_3\} \\
&= \{q_0, q_1, q_2, q_3\} = \varphi(\widehat{q}_3).
\end{aligned}$$

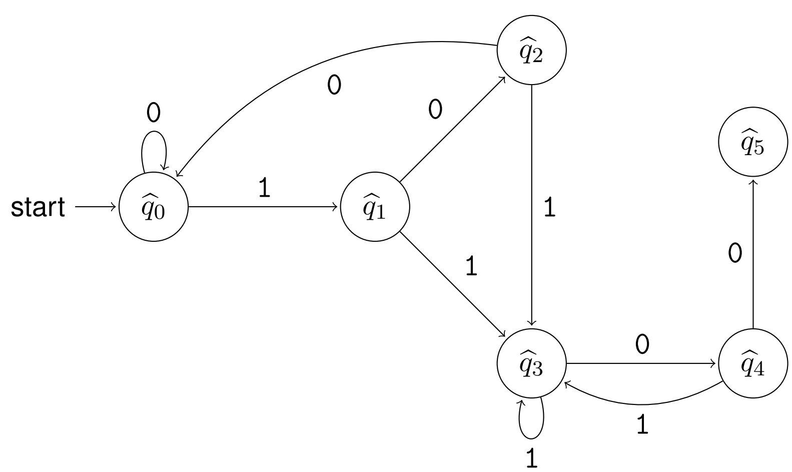
Thus $\widehat{\delta}(\widehat{q}_4, 1) = \widehat{q}_3$. Since $\widehat{q}_3$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Completing the Fifth Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the fifth time $\widehat{q}_4$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0, \widehat{q}_1, \widehat{q}_2, \widehat{q}_3, \widehat{q}_4\} \quad \text{and} \quad \widehat{R} = \{\widehat{q}_5\}$$

at the end of the of the fifth execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Determining Transitions out of $\widehat{q}_5$

Since $\widehat{R}$ is not empty the test at line 5 of the algorithm is passed, the sixth time it is checked, and the body of the `while` loop (at lines 6–17) is executed for a sixth time.

Now, since $\widehat{R}$ only includes one state — $\widehat{q}_5$ — when the step at line 6 is reached and executed, it is necessary to set $\widehat{q}$ to be $\widehat{q}_5$ at this point. In this case $V = \varphi(\widehat{q}_5) = \{q_0, q_3\}$.

The step at line 7 and the inner loop at lines 8–15 are used next to compute $\widehat{\delta}(\widehat{q}_5, \sigma)$ for $\sigma = 0$ and for $\sigma = 1$.

Determining $\widehat{\delta}(\widehat{q}_5, 0)$

Since $V = \{q_0, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_0 = 0$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0, q_3\}} \left(\bigcup_{s \in \delta(r, 0)} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, 0)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_3, 0)} \mathit{Cl}_\lambda(s) \\ &= \mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_3) \\ &= \{q_0\} \cup \{q_3\} \\ &= \{q_0, q_3\} = \widehat{q}_5. \end{aligned}$$

Thus $\widehat{\delta}(\widehat{q}_5, 0) = \widehat{q}_5$. Since $\widehat{q}_5$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Determining $\widehat{\delta}(\widehat{q}_5, 1)$

Since $V = \{q_0, q_3\}$ and $\sigma_{\mathtt{j}} = \sigma_1 = 1$, the next execution of the step at line 9 computes W as follows:

$$\begin{aligned} W &= \bigcup_{r \in \{q_0, q_3\}} \left(\bigcup_{s \in \delta(r, 1)} \mathit{Cl}_\lambda(s) \right) \\ &= \bigcup_{s \in \delta(q_0, 1)} \mathit{Cl}_\lambda(s) \quad \cup \bigcup_{s \in \delta(q_3, 1)} \mathit{Cl}_\lambda(s) \\ &= (\mathit{Cl}_\lambda(q_0) \cup \mathit{Cl}_\lambda(q_1)) \cup \mathit{Cl}_\lambda(q_3) \\ &= (\{q_0\} \cup \{q_1, q_2\}) \cup \{q_3\} \\ &= \{q_0, q_1, q_2, q_3\} = \varphi(\widehat{q}_3). \end{aligned}$$

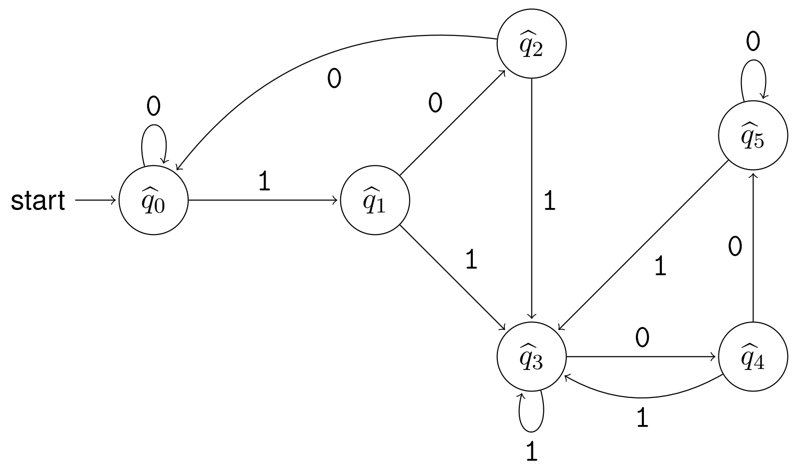
Thus $\widehat{\delta}(\widehat{q}_5, 1) = \widehat{q}_3$. Since $\widehat{q}_3$ already belongs to $\widehat{Q} \cup \widehat{R}$, $\widehat{R}$ is not changed by this execution of the body of the inner loop.

Completing the Sixth Execution of the Body of the Outer Loop

When the steps at at lines 16 and 17 are executed for the sixth time $\widehat{q}_5$ is moved from $\widehat{R}$ to $\widehat{Q}$, so that

$$\widehat{Q} = \{\widehat{q}_0, \widehat{q}_1, \widehat{q}_2, \widehat{q}_3, \widehat{q}_4, \widehat{q}_5\} \quad \text{and} \quad \widehat{R} = \emptyset$$

at the end of the of the sixth execution of the body of the outer loop. The partial deterministic finite automaton, which has been obtained so far, is as follows.



Choosing the Set of Accepting States

Finally, for each state $\widehat{q} \in \widehat{Q}$, one should include $\widehat{q}$ in the set $\widehat{F}$ ***if and only if*** $\varphi(\widehat{q}) \cap F \neq \emptyset$. Nor the deterministic finite automaton whose states and transitions have been identified,

- $\varphi(\widehat{q}_0) \cap F = \{q_0\} \cap \{q_3\} = \emptyset$, so $\widehat{q}_0 \notin \widehat{F}$.
- $\varphi(\widehat{q}_1) \cap F = \{q_0, q_1, q_2\} \cap \{q_3\} = \emptyset$, so $\widehat{q}_1 \notin \widehat{F}$.
- $\varphi(\widehat{q}_2) \cap F = \{q_0, q_2\} \cap \{q_3\} = \emptyset$, so $\widehat{q}_2 \notin \widehat{F}$.
- $\varphi(\widehat{q}_3) \cap F = \{q_0, q_1, q_2, q_3\} \cap \{q_3\} = \{q_3\}$, so $\widehat{q}_3 \in \widehat{F}$.
- $\varphi(\widehat{q}_4) \cap F = \{q_0, q_2, q_3\} \cap \{q_3\} = \{q_3\}$, so $\widehat{q}_4 \in \widehat{F}$.
- $\varphi(\widehat{q}_5) \cap F = \{q_0, q_3\} \cap \{q_3\} = \{q_3\}$, so $\widehat{q}_5 \in \widehat{F}$.

Thus $\widehat{F} = \{\widehat{q}_3, \widehat{q}_4, \widehat{q}_5\}$.

The Deterministic Finite Automaton

The resulting DFA is as follows.