Lecture #1: Introduction to Deterministic Finite Automata
Key Concepts

Definition 1. A ***deterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite, nonempty, set of ***states***;
- Σ is an ***alphabet*** such that $Q \cap \Sigma = \emptyset$;
- $\delta : Q \times \Sigma \to Q$ is a ***transition function***;
- $q_0 \in Q$ is the ***start state***; and
- $F \subseteq Q$ is the set of ***accept states***.

In particular, δ is a ***total*** function from $Q \times \Sigma$ to Q.

Definition 2. Consider a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$. The ***extended transition function*** of M is a function

$$\delta^\star : Q \times \Sigma^\star \to Q$$

describing the state that can be reached from a given state when processing a *string*: For every state $q \in Q$ and every string $\omega \in \Sigma^\star$,

$$\delta^\star(q, \omega) = \begin{cases} q & \text{if } \omega = \lambda, \\ \delta(\delta^\star(q, \mu), \sigma) & \text{if } \omega = \mu \cdot \sigma \text{ for a string } \mu \in \Sigma^\star \text{ and symbol } \sigma \in \Sigma. \end{cases}$$

Definition 3. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite automaton and let $\omega \in \Sigma^\star$. Then M ***accepts*** ω if $\delta^\star(q_0, \omega) \in F$, and M ***rejects*** ω otherwise.

Definition 4. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite automaton. The ***language of*** $\boldsymbol{M}$, $L(M)$, is the set

$$\{\omega \in \Sigma^\star \mid M \text{ accepts } \omega\}.$$

Definition 5. Let Σ be an alphabet. Then a language $L \subseteq \Sigma^\star$ is a ***regular language*** if L is the language, $L(M)$, of some deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ (with the same alphabet Σ).