Lecture #1: Introduction to CPSC 351
Introduction to Deterministic Finite Automata

What To Do Before the Lecture

1. Please look at the material in the "Introduction and Discrete Mathematics Review" part of the course web site. If you have time, watch the "Welcome to CPSC 351!" video that is included here: You should be able to understand it (if time is short) if you watch it at double speed.
2. Skim through the "Mathematics Review" material to make sure that everything is familiar (or easy to understand, if anything is new).
3. Work through either the (slightly longer) "Alphabets, Strings and Languages" document or the "Key Concepts" document found immediately after it on the course web site. The longer document includes a few examples that the "Key Concepts" document does not have, but the "Key Concepts" document includes all of the definitions and notation that will be needed, later on.
4. Look at the beginning of the material in the "Finite Automata and Regular Languages" part of the course web site — up to the beginning of the material for Lecture #1.
5. Watch the video for Lecture #1 — noting that it will probably be understandable if you play it at double speed. If you do not have time for this then look at the "Key Concepts" document that is found, immediately after the video for this lecture, on the course web site, instead.
6. ***Print*** and read through the rest of this document and — if you have time — try to solve the problems! These should help you to make sure that you understand how deterministic finite automata process strings, and use proof techniques — that you learned about in CPSC 251 or MATH 251 — to prove things about these automata.

Lecture Presentation

A Brief Introduction to CPSC 351

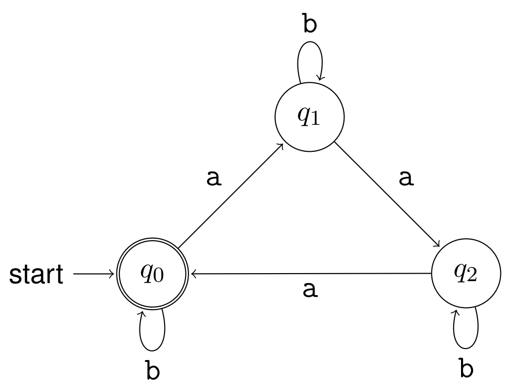
The instructor will start by briefly discussing the learning goals, topics, and organization of this course.

Introduction to Deterministic Finite Automata

Note: It will be assumed that you have completed the preparatory viewing (or reading) for this lecture — this material will not be introduced during the lecture presentation!

Problems To Be Solved

Consider a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that has alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and that can be represented as follows. One can see from this picture that $Q = \{q_0, q_1, q_2\}$, q_0 is the start state (as the representation of M as a 5-tuple also shows) and that $F = \{q_0\}$.



1. Consider the ***transition function*** $\delta : Q \times \Sigma \to Q$ for this deterministic finite automaton.

   (a) Complete this specification of this function.

$$\delta(q_0, \mathtt{a}) = \qquad \delta(q_1, \mathtt{a}) = \qquad \delta(q_2, \mathtt{a}) =$$

$$\delta(q_0, \mathtt{b}) = \qquad \delta(q_1, \mathtt{b}) = \qquad \delta(q_2, \mathtt{b}) =$$

(b) Give a ***transition table*** for this deterministic finite automaton.

2. Consider the string $\omega = \mathtt{abaabb}$.

(a) Complete the following sequence to compute $\delta^\star(q_0, \omega)$.

$$\delta^\star(q_0, \lambda) =$$

$$\delta^\star(q_0, \mathtt{a}) =$$

$$\delta^\star(q_0, \mathtt{ab}) =$$

(b) Complete the following derivation of $\delta^\star(q_0, \omega)$ (which uses the definition of the extended transition function directly).

$$\begin{aligned}\delta^\star(q_0, \omega) &= \delta^\star(q_0, \mathtt{abaabb}) \\ &= \delta(\delta^\star(q_0, \mathtt{abaab}), \mathtt{b}) \\ &= \end{aligned}$$

3. What are the states q_0, q_1 and q_2 used to keep track of, as symbols in an input string are being processed?

4. Let $S_0, S_1, S_2 \subseteq \Sigma^\star$ be as follows.

- $S_0 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 0 modulo $3\}$
- $S_1 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 1 modulo $3\}$
- $S_2 = \{\omega \in \Sigma^\star \mid$ the number of copies of "a" in ω is congruent to 2 modulo $3\}$

Consider the following.

Claim 1. *Consider the above deterministic finite automaton M, and the above sets S_0, S_1, and S_2. The following properties are satisfied for every string $\omega \in \Sigma^\star$.*

(a) $\delta^\star(q_0, \omega) = q_0$ *if and only if* $\omega \in S_0$.
(b) $\delta^\star(q_0, \omega) = q_1$ *if and only if* $\omega \in S_1$.
(c) $\delta^\star(q_0, \omega) = q_2$ *if and only if* $\omega \in S_2$.

The goal, for the rest of this question, is to prove this claim.

(a) Describe, as precisely as you can, a ***proof technique*** (which you studied in CPSC 251 or MATH 271) which could be used to prove this claim.

(b) Ideally, you answered the above question by mentioned some form of ***mathematical induction*** — identifying both the form of mathematical induction to be used, and what you would be inducting on.

To continue, state (as precisely as you can) ***what you need to prove*** in order to complete the ***basis*** for this proof.

(c) Given the outline of the ***inductive step***. As part of this, you should introduce a new parameter (possibly called k) that is to be used. You should also state *what can be assumed* and *what must be proved* in the inductive step.

(d) Using the above information, prove the claim.