Lecture #22: Application — Analysis of Algorithms Lecture Presentation

Review of Preparatory Material

Analyzing a Randomized Algorithm

The lecture presentation included a *randomized algorithm* to check whether an input integer key was stored in an input integer array A — as given in Figure 2 on the next page. This algorithm calls a variant of a "Linear Search" algorithm as a subroutine — as shown in Figure 1 on the next page, as well.

The goal for this initial problem is to analyze the *running time* of this randomized algorithm when its input includes an integer array A with length n.

Number of Steps Used by the Deterministic Subroutine:

```
integer dSearch (integer[] A, integer key) {
1. integer n := A.length
2. integer i := 0
3. while (i < n) {
4.
     if(A[i] == key) {
5.
       return true
     }
6.
     i := i + 1
   }
7. return false
}
                 Figure 1: Subroutine Implementing Linear Search
boolean rSearch3 (integer[] A, integer key) {
1. integer n := A.length
2. integer i := 0
3. while (i < n) {
     Choose j uniformly from the set \{0,1,2,\dots,\mathtt{n}-1\} — independently from any
     previous selections.
5.
     if(A[j] == key) {
6.
       return true
     }
7. i := i + 1
```

Figure 2: Randomized Algorithm for Searching in an Integer Array

}

}

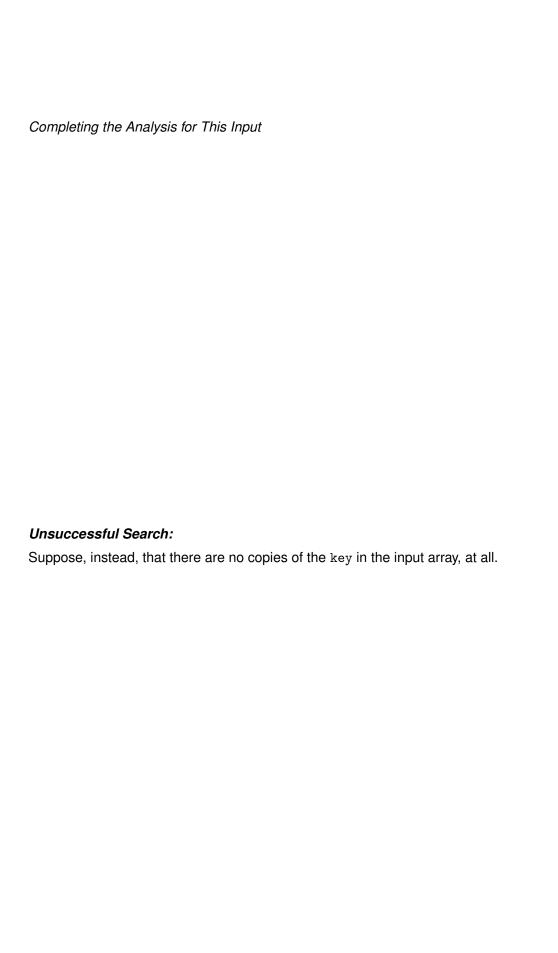
8. return dSearch(A, key)

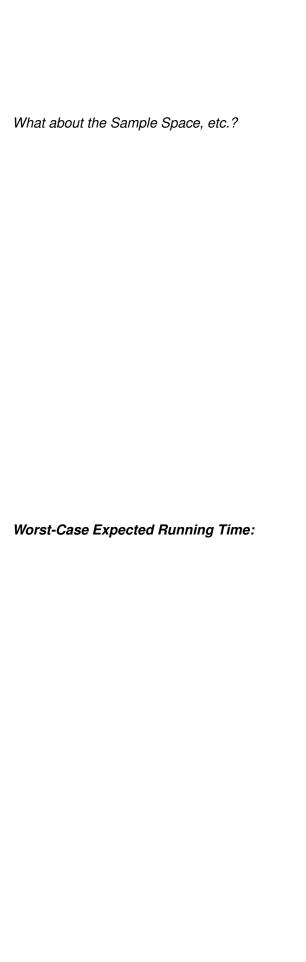
Successful Search:

Let k be an integer such that $1 \le k \le n$ and suppose that exactly k copies of the input key are stored in the array A. Suppose, as well, that the *first* copy of the key is stored at index ℓ — so that ℓ is an integer such that $0 \le \ell \le n-k$.

Defining the Sample Space







1 Dealing with a Complicated Expression

This kind of analysis can result in complicated expressions, whose value we wish to give in "closed form". If we cannot give a simplified version of the expression's value then we might be able to work with a simplified form of an *approximation*, *upper bound* or *lower bound* for this expression, instead.

One such expression was found in the preparatory reading — where the expected value of a random variable was found to be

$$\mathsf{E}[T] = \sum_{i=0}^{n-1} \left(1 - \frac{k}{n}\right)^i \cdot \frac{k}{n} \cdot (4i + 6) + \left(1 - \frac{k}{n}\right)^n \cdot (4n + 3).$$

It is doubtful that this kind of expression will have a simple solution — but it is possible to approximate its value.

An Easy Case: What is this when k = 0?

Another Easy Case: What is this when k = n?

Now suppose, instead, that k is an integer such that $1 \leq k \leq n-1.$

Making a Start:

Now, what can we do with the expression $\left(1-\frac{k}{n}\right)^n$?