CPSC 351 — Tutorial Exercise #13

Oracle Reductions

About This Exercise

These questions are intended to give you practice in establishing ***oracle reductions*** between languages. They are of the difficulty, and length, that would be appropriate for questions on a ***test*** in CPSC 351.

Problems To Be Solved

1. Let Σ be an alphabet, let $L \subseteq \Sigma^\star$, and consider the language

   $$L \circ L = \{\omega_1 \cdot \omega_2 \mid \omega_1, \omega_2 \in L\} \subseteq \Sigma^\star.$$

   Prove that $L \circ L \preceq_{\mathsf{O}} L$.

2. Let Σ be an alphabet. The ***reversal***, ω^R, of a string $\omega \in \Sigma^\star$, is the string obtained by reversing the order of the symbols in the string. That is, if

   $$\omega = \alpha_1 \alpha_2 \dots \alpha_{n-1} \alpha_n$$

   where $\alpha_1, \alpha_2, \dots \alpha_{n-1}, \alpha_n \in \Sigma$ (so that n is the length of ω), then

   $$\omega^R = \alpha_n \alpha_{n-1} \dots \alpha_2 \alpha_1.$$

   (a) Show that the function $f : \Sigma^\star \to \Sigma^\star$ such that $f(\omega) = \omega^R$, for every string $\omega \in \Sigma^\star$, is a computable function.

   Let $L \subseteq \Sigma^\star$. Let L^R be the set of reversals of strings in L, that is,

   $$L^R = \{\omega^R \mid \omega \in L\}.$$

   (b) Prove that $L^R \preceq_{\mathsf{O}} L$.

   (c) Could the above result be used to prove that if L is ***undecidable*** then L^R is ***undecidable*** as well? If the answer if "no" then what reduction(s) could you give, to prove this, instead?