

Lecture #15: Many-One Reductions

Lecture Presentation

Review of Preparatory Material

Problems To Be Solved

1. Recall that an alphabet

$$\Sigma_D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

was introduced. Following the definitions of the **unpadded decimal representation** of a positive integer n , the **unpadded decimal representation** of a natural number n as a string in Σ_D^* , the set $\text{Unpad} \subseteq \Sigma_D^*$ of unpadded decimal representations of natural numbers, a (possibly) **padded decimal representation** of a natural number n , and the set $\text{Pad} \subseteq \Sigma_D^*$ of (possibly) padded decimal representations of natural numbers, a set $S \subseteq \mathbb{N}$ was introduced and used to define **two** languages, $\mathcal{U}_S \subseteq \text{Unpad} \subseteq \Sigma_D^*$ and $\mathcal{P}_S \subseteq \text{Pad} \subseteq \Sigma_D^*$:

- $\mathcal{U}_S \subseteq \text{Unpad}$ is the set of all **unpadded decimal representations** of numbers $n \in S$.
- $\mathcal{P}_S \subseteq \text{Pad}$ is the set of all **padded decimal representations** of numbers $n \in S$.

Our goal is to prove that $\mathcal{U}_S \preceq_M \mathcal{P}_S$ for every subset $S \subseteq \mathbb{N}$.

What We Need To Provide — and the Properties It Must Satisfy:

Let $\omega \in \Sigma_D^*$. ***What Can We Set $f(\omega)$ To Be, When $\omega \notin \text{Unpad}$? Why?***

What Can We Set $f(\omega)$ To Be, When $\omega \in \text{Unpad}$ Why?

The Function f :

A First Claim about f and Its Proof:

A Second Claim About f and Its Proof:

A Third Claim About f and Its Proof:

2. The lecture notes introduced a language $\text{HALT}_{\text{TM}} \subseteq \text{TM+I}$ and a proof that

$$\text{HALT}_{\text{TM}} \preceq_{\text{M}} \text{A}_{\text{TM}}.$$

Recall that HALT_{TM} was the set of encodings of Turing machines

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

and input string $\omega \in \Sigma^*$ such that M 's execution on the input string ω *halts*, while A_{TM} is another subset of TM+I , namely the set of encodings of Turing machines M (as above) and input strings $\omega \in \Sigma^*$ such that M *accepts* ω .

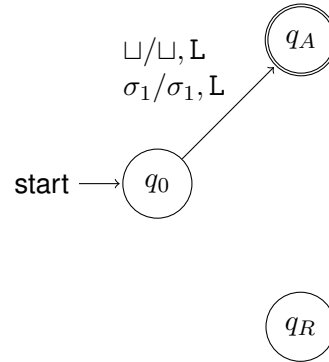
Suppose we wish to prove that $\text{A}_{\text{TM}} \preceq_{\text{M}} \text{HALT}_{\text{TM}}$. ***What do we need to provide — and what properties must it satisfy?***

Let $f_1 : \Sigma_{\text{TM}}^* \rightarrow \Sigma_{\text{TM}}^*$ such that $f_1(\mu) = \mu$ for every string $\mu \in \Sigma_{\text{TM}}^*$.

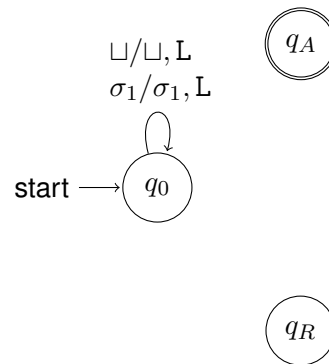
Is f_1 a Many-One Reduction from A_{TM} to HALT_{TM} ?

Why — or Why Not?

Next let us consider a pair of Turing machines with input alphabet $\Sigma = \{\sigma_1\}$ and tape alphabet $\Gamma = \{\sigma_1, \sqcup\}$. The first of these Turing machines, M_Y , is as follows:



The second of these Turing machines, M_N , is as follows:



In both of these pictures the accept state is shown as “ q_A ” instead of q_{accept} , and the reject state is shown as “ q_R ” instead of “ q_{reject} ”, to simplify the picture.

Now note that if $x_{\text{Yes}} \in \Sigma_{\text{TM}}^*$ is the encoding of the above Turing machine M_Y , and the empty string λ , then $x_{\text{Yes}} \in \text{HALT}_{\text{TM}}$. On the other hand, if $x_{\text{No}} \in \Sigma_{\text{TM}}^*$ is the encoding of the Turing machine M_N , and the empty string λ , then $x_{\text{no}} \notin \text{HALT}_{\text{TM}}$.

Let $f_2 : \Sigma_{\text{TM}}^* \rightarrow \Sigma_{\text{TM}}^*$ such that, for every string $\mu \in \Sigma_{\text{TM}}^*$,

$$f_2(\mu) = \begin{cases} x_{\text{Yes}} & \text{if } \mu \in A_{\text{TM}}, \\ x_{\text{No}} & \text{if } \mu \notin A_{\text{TM}}. \end{cases}$$

Is f_2 a Many-One Reduction from A_{TM} to HALT_{TM} ?

Why — or Why Not?

One More Attempt:

