

Lecture #14: Oracle Reductions

What Will Happen During the Lecture

Goals of this lecture presentation will be to help students understand **oracle reductions**, how to show that these exist, and how to use these reductions to prove that languages are undecidable.

Problems To Be Solved

Two problems will be solved. If you have time then you should try to solve the first one after you have completed the preparatory reading for this lecture but before you attend the lecture presentation.

1. Let $\Sigma = \{a, b, c\}$, let $L \subseteq \Sigma^*$, and let

$$L_a = \{\omega \in \Sigma^* \mid \omega \in L \text{ and } \omega \text{ ends with "a"}\}.$$

Prove that $L_a \preceq_O L$.

2. Consider the language $\text{LOOP}_{\text{TM}} \subseteq \Sigma_{\text{TM}}^*$, including encodings of Turing machines M and input strings ω for M such that M **loops** on ω .

Establish an oracle reduction, involving LOOP_{TM} and some other language, that can be used to prove that the language LOOP_{TM} is undecidable.