Lecture #14: Oracle Reductions

Lecture Presentation

Review of Preparatory Material

A First Problem — Suitable for a Test

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, let $L \subseteq \Sigma^\star$, and let

$$L_a = \{\omega \in \Sigma^\star \mid \omega \in L \text{ and } \omega \text{ ends with “a”}\}.$$

You were asked to prove that $L_a \preceq_{\mathsf{O}} L$.

What Do You Need To Provide?

On input $\omega \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^\star$:

1. `if` (ω ends with "a") `{`
2. `if` ($\omega \in L$) `{`
3. `accept` ω
   `} else {`
4. `reject` ω
   `}`
   `} else {`
5. `reject` ω
   `}`

Figure 1: Algorithm Used for an Oracle Reduction

Details of Reduction

Consider the algorithm shown in Figure 1,

Establishing Correctness

Implementation Details

Why is This “Suitable as a Test Problem”?

A Second Problem — Suitable for an Assignment

Consider the language $\text{LOOP}_{\text{TM}} \subseteq \Sigma_{\text{TM}}^{\star}$, including encodings of Turing machines M and input strings ω for M such that M ***loops*** on ω.

You were asked to establish an oracle reduction, involving LOOP_{TM} and some other language, that can be used to prove that the language LOOP_{TM} is undecidable.

What We Need To Provide:

Useful Information That We Already Have:

At this point in the course several similar languages have been considered:

- The language $\text{TM+I} \subseteq \Sigma_{\text{TM}}^{\star}$ of encodings of Turing machines M and input strings ω for M. This language is ***decidable*** and it follows from the definitions of these languages that $\text{LOOP}_{\text{TM}} \subseteq \text{TM+I} \subseteq \Sigma_{\text{TM}}$.

- The language $\text{A}_{\text{TM}} \subseteq \Sigma_{\text{TM}}^{\star}$ of encodings of Turing machines M and input strings ω for M such that M accepts ω.

  This language is ***recognizable***: A multi-tape Turing machine with language A_{TM} (called a "universal Turing machine") was described in Lecture #13 — and it follows, by results about multi-tape Turing machines included in Lecture #12, that there must also exist a standard (single tape) Turing machine, $M_{\text{A}_{\text{TM}}}$, whose language is A_{TM}, as well.

  On the other hand it was proved in Lecture #13, that the language A_{TM} is ***undecidable***.

Which Oracle Reduction Will We Present, To Solve This Problem?

Details of Reduction: An Algorithm To Consider

Details of Reduction: Implementation Details

Why is This More Appropriate for an Assignment?