

Lecture #13: Universal Turing Machines

What Will Happen During the Lecture

Goals of this lecture presentation will be to help students understand **universal Turing machines** and why they are important.

The preparatory material introduced an alphabet, Σ_{TM} , which were used to define **encodings of Turing machines** — as strings in Σ_{TM}^* — along with four languages:

- $L_{\text{TM}} \subseteq \Sigma_{\text{TM}}^*$ — the language of encodings of Turing machines. It was noted, in the preparatory material for the lecture, that the language L_{TM} is **decidable**.
- $L_{\text{TM}+I} \subseteq \Sigma_{\text{TM}}^*$ — the language of encodings of Turing machines M and input strings ω for M . It was noted, in the preparatory material for the lecture, that the language $L_{\text{TM}+I}$ is also **decidable**.
- $A_{\text{TM}} \subseteq L_{\text{TM}+I}$ — the language of encodings of Turing machines M , and input strings ω for M , such that M **accepts** ω . Since A_{TM} is the language of the “universal Turing machine”, introduced in the preparatory material, A_{TM} is **recognizable**. However, a proof was sketched, in the preparatory material, that A_{TM} is **undecidable**.
- $A_{\text{TM}}^C = \{\mu \in \Sigma_{\text{TM}}^* \mid \mu \notin A_{\text{TM}}\} \subseteq \Sigma_{\text{TM}}^*$ — the **complement** of the language A_{TM} . A proof was sketched, in the preparatory material, that A_{TM}^C is **unrecognizable**.

During the lecture presentation, the proofs that “ A_{TM} is undecidable” and that “ A_{TM}^C is unrecognizable” will be explored, in a bit more detail — and students will have the opportunity to ask questions about these results.