Lecture #11: Simulations and Simple Turing Machine Variants

Lecture Presentation

Review of Preparatory Material

Turing Machines with Two-Way Infinite Tapes

A New Turing Machine Variant

A ***Turing machine with a two-infinite tape*** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input — and the tape head is initially pointing to the leftmost symbol in the input if the input is nonempty. Computation is as usual, except that the tape head never encounters an end of the tape as it moves left.

This kind of Turing machine can also be modelled as a 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

where Q, Σ, Γ, δ, q_0, q_{accept} and q_{reject} are exactly as described as for a standard Turing machine. The most significant difference is that, since there is no "leftmost cell" of the tape, an attempt to move left, past the initial position of the tape head, will be successful — with a copy of "$\sqcup$" being visible, when this attempt (initially) succeeds.

A Reduction in One Direction

Claim. *Let $L \subseteq \Sigma^\star$ for an alphabet Σ. If L is Turing-recognizable — so that there is a standard (one-tape) Turing machine M with input alphabet Σ and language L. Then there exists a Turing machine with a two-way infinite tape, $\widehat{M}$ with alphabet Σ such that $L(\widehat{M}) = L$ as well. Furthermore, if M decides L then $\widehat{M}$ decides L, as well.*

Using $\widehat{M}$ to Represent a Configuration of M

Initialization

Simulating a Move of M

Completing the Proof

Reduction in Another Direction

Claim. *Let $L \subseteq \Sigma^\star$ for an alphabet Σ. If there exists a Turing machine with a two-way infinite tape, M, with input alphabet Σ and language L, then L is Turing-recognizable — so that there exists a (standard) Turing machine $\widehat{M}$ with alphabet Σ such that $L(\widehat{M}) = L$ as well. Furthermore, if M decides L then $\widehat{M}$ decides L, as well.*

Using M to Represent a Configuration of M

Initialization

Simulating a Move of M

Completing the Proof