Lecture #10: Introduction to Turing Machines
Describing a Turing Machine's Moves in More Detail

This document describes how configurations of a deterministic Turing machine

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

are updated, during steps of a Turing machine's computation, in a bit more detail. It should be consulted if the preparatory material and lecture presentation for Lecture #10 was not detailed enough for this to be understood.

Suppose that M is in the configuration represented by a string

$$\omega_1 \, q \, \omega_2$$

for a state $q \in Q$ and for strings $\omega_1, \omega_2 \in \Gamma^\star$.

If $q \in \{q_{\text{accept}}, q_{\text{reject}}\}$ then M's computation has already halted — and there is no next step that can be taken. Suppose, instead, that $q \in Q \setminus \{q_{\text{accept}}, q_{\text{reject}}\}$ (that is, $q \in Q$ but $q \notin \{q_{\text{accept}}, q_{\text{reject}}\}$).

Let σ be the leftmost symbol in ω_2, if ω_2 is not the empty string, and let $\sigma = \sqcup$ otherwise. Then $\sigma \in \Gamma$ is the symbol that is currently visible on M's tape. Furthermore, let $\mu_2 \in \Gamma^\star$ such that $\omega_2 = \sigma\mu_2$ if $\omega_2 \neq \lambda$, and let $\mu_2 = \lambda$ if $\omega_2 = \lambda$.

Since $q \notin \{q_{\text{accept}}, q_{\text{reject}}\}$ and $\sigma \in \Gamma$ the transition function δ is defined for the ordered pair (q, σ): Suppose that

$$\delta(q, \sigma) = (r, \tau, d)$$

where $r \in Q$, $\tau \in \Gamma$, and $d \in \{\mathtt{L}, \mathtt{R}\}$. Then either $d = \mathtt{L}$ or $d = \mathtt{R}$; these cases are considered separately, below.

- *Case:* $d = \mathtt{L}$.

  Either $\omega_1 = \lambda$, in which case M's tape head is pointing to the leftmost cell of the tape, or $\omega_1 \neq \lambda$, in which case M's tape head points to a cell that is farther to the right. These cases should also be considered separately.

– *Subcase:* $\omega_1 = \lambda$, so that M's tape head is pointing to the leftmost cell of the tape. In this case it is not possible to move M's tape head farther to the left, at all, so that the position of M's tape head does not change. The symbol "σ" should be replaced by the symbol "τ".

  Now, if either $\tau \neq \sqcup$ or $\mu_2 \neq \lambda$ then the string "$\tau\mu_2$" does not end with "$\sqcup$", and the configuration that M moves to is represented by the string

$$r\,\tau\mu_2\,.$$

  On the other hand, if $\tau = \sqcup$ and $\mu_2 = \lambda$ then the $\tau\mu_2 = \sqcup$, which ends with a blank, and the configuration that M moves to is represented by the string

$$r\,.$$

– *Subcase:* $\omega_1 \neq \lambda$, so that M's tape head is pointing to a cell that is farther to the right than the leftmost cell of the tape. In this case it *is* possible to move M's tape head farther to the left, so that the position of M's tape head should change. Suppose that

$$\omega_1 = \mu_1\gamma$$

  for $\mu_1 \in \Gamma^\star$ and $\gamma \in \Gamma$.

  If either $\mu_2 \neq \lambda$ or $\tau \neq \sqcup$ (or both) then the configuration that M moves to is represented by the string

$$\mu_1\,r\,\gamma\tau\mu_2$$

  — because the machine has replaced a copy of σ on the tape[1] with a copy of τ and then moved the tape head one position to the left.

  On the other hand, if $\mu_2 = \lambda$ and $\tau = \sqcup$ then either $\gamma = \sqcup$ or $\gamma \neq \sqcup$. If $\gamma \neq \sqcup$ then (after this step is taken) γ is the rightmost non-blank symbol on the tape and the configuration that M moves to is represented by a string

$$\mu_1\,r\,\gamma\,.$$

  Finally, if $\gamma = \sqcup$ then there are now no non-blank symbols at, or to the right of, the position of the tape head at all, and the configuration that M moves to is represented by a string

$$\mu_1\,r\,.$$

• *Case:* $d = \mathtt{R}$.

  Since a move to the right on the tape is always possible, M's tape head should be moved to the right, by one position, as the transition is applied. The transition that M moves to is represented by a string

$$\omega_1\tau\,r\,\mu_2\,.$$

[1] or a copy of $\sqcup$, if $\omega = \lambda$