CPSC 351 — Tutorial Exercise #9
Nonregular Languages, Part Two

1 About This Exercise

This exercise is intended to help you to understand and use closure properties to prove that languages are not regular.

Getting Started

This initial question concerns a ***mistake*** that students often make when they try to use the techniques used in this exercise. There will probably not time to discuss this problem during the tutorial — but you should discuss these with the instructor, during the instructor's office hours, if you have questions about them.

1. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and let $L = \emptyset$. Please identify the ***error*** that has been made, in the following incorrect proof, as precisely as you can. (This error is similar to one that students have made, when writing proofs that languages are not regular, in the past.)

   Claim: The above language, L, is not regular.

   Proof. Recall that the languages

   $$L_1 = \{\mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{Z} \text{ and } n \geq 0\}$$

   and

   $$L_2 = \{\mathtt{a}^n\mathtt{b}^m \mid n \in \mathbb{Z},\ m \in \mathbb{Z},\ n, m \geq 0 \text{ and } n \neq m\}$$

   are languages such that $L_1 \subseteq \Sigma^\star$, $L_2 \subseteq \Sigma^\star$, L_1 is not regular, and L_2 is not regular. Since $L_1 \cap L_2 = \emptyset = L$ it follows that the language $L = \emptyset$ is not regular, as well. □

Problems To Be Solved in the Tutorial

2. Let $\Sigma = \{\mathtt{a}\}$ and recall, from the previous exercise, that the language

$$L_p = \{\mathtt{a}^n \mid n \text{ is a prime number}\} \subseteq \Sigma^\star$$

is *not* a regular language. Recall, as well, that a positive integer n is ***composite*** if $n \geq 2$ and there exist integers k and ℓ such that $2 \leq k, \ell \leq n-1$ and $k \times \ell = n$. Every positive integer n, such that $n \geq 2$, is either prime or composite — but not both. On the other hand, the integers 0 and 1 are neither prime nor composite.

Let $L_c \subseteq \Sigma^\star$ be the language

$$L_c = \{\mathtt{a}^n \mid n \text{ is composite}\}.$$

Use this information, and one or more of the ***closure properties*** for regular languages that have now been introduced in this course, to prove that L_c is *not* a regular language.