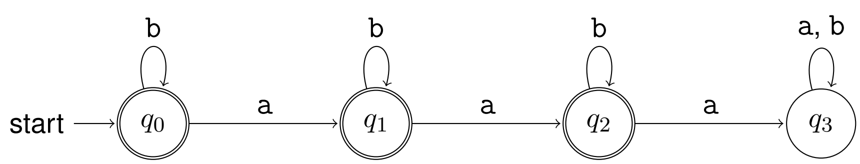
CPSC 351 — Tutorial Exercise #2
Introduction to Deterministic Finite Automata

About This Exercise

The goal of this exercise is to help you to understand how deterministic finite automata can be described, how they are used to process strings of symbols, and how the languages of deterministic finite automata are defined.

If you have time: Please try to solve the problems in this exercise ***before*** attending the tutorial where it will be discussed.

The exercise concerns a deterministic finite automaton M that has alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and that can be represented as follows.



Getting Started

The first four problems should be straightforward, if you understood the introduction to deterministic finite automata given in Lecture #2. Discussion of these will probably be limited, in the tutorial, so that there is time to discuss the (somewhat) more challenging exercises that follow them.

1. Give the set Q of ***states*** in M and identify the ***start state***.

2. Give the set F of ***accepting states*** in M.

3. Describe the ***transition function*** $\delta : Q \times \Sigma \to Q$ by completing the following ***transition table***.

	a	b
q_0		
q_1		
q_2		
q_3		

4. Trace the execution of M on each of the following input strings — listing the sequence of states that are visited as symbols in the string are seen and processed, and stating whether the string is in the language of M.

   (a) λ
   (b) `a`
   (c) `b`
   (d) `ab`
   (e) `ba`
   (f) `abbab`
   (g) `aabbab`
   (h) `aaaabbb`

Problems Discussed in the Tutorial

5. Give a ***brief*** description, in simple English, for each of the following subsets of $\Sigma^\star$.

   (a) $\{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_0\}$
   (b) $\{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_1\}$
   (c) $\{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_2\}$
   (d) $\{\omega \in \Sigma^\star \mid \delta^\star(q_0, \omega) = q_3\}$

   How could you ***prove*** that your answers are correct (using proof techniques that you learned about in CPSC 251 or MATH 271)?

6. Use your answer for the previous question to give a ***brief*** description, in simple English, of the language of (this particular DFA) M.

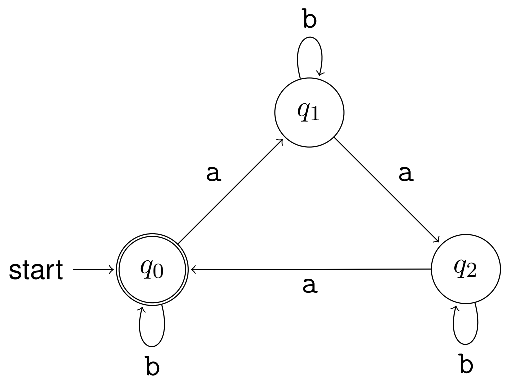


Figure 1: Deterministic Finite Automaton Considered in Lecture #2

What Will Happen During the Tutorial: Once again, if possible, students will form groups based on how far they got. The teaching assistant will visit groups, as time allows, to help move discussions along — noting questions that were hard to answer, in the case the instructor can clarify things later.

A More Challenging Problem

The will probably not be time for this to be discussed during the tutorial. However, this problem might be useful for students needing more practice using mathematical induction to solve problems — and it can be discussed with the instructor during office hours.

Recall that the lecture presentation for Lecture #2 included a consideration of the ***deterministic finite automaton*** $M = (Q, \Sigma, \delta, q_0, F)$ such that $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, q_0 is the start state, $F = \{q_0\}$, and the transition function $\delta : Q \times \Sigma \to Q$ is as shown in Figure **??**, above.

During the lecture presentation, the following claim was introduced.

Theorem 1. *Let $n \in \mathbb{N}$. Then, for every string $\omega \in \Sigma^\star$ such that $|\omega| = n$, the following properties are satisfied.*

(a) $\delta^\star(q_0, \omega) = q_0$ *if and only if the number of copies of "*$\mathtt{a}$*" in* ω *is congruent to* 0 *mod* 3.

(b) $\delta^\star(q_0, \omega) = q_1$ *if and only if the number of copies of "*$\mathtt{a}$*" in* ω *is congruent to* 1 *mod* 3.

(c) $\delta^\star(q_0, \omega) = q_2$ *if and only if the number of copies of "*$\mathtt{a}$*" in* ω *is congruent to* 2 *mod* 3.

It is possible to prove this using mathematical induction, in particular, by induction on n, using the standard form of mathematical induction.

7. In this question you will be asked to give some of the details of the proof of this claim.

   (a) Complete the ***basis*** for this proof.

   (b) The ***Inductive Claim***, which is to be proved in the Inductive Step, has three parts ("part (a)", "part (b)", and "part (c)"), just like the claim that is being proved. Give the proof of part (a) of the Inductive Claim.

   (c) The proofs of parts (b) and (c) of the Inductive Claim are very similar to the proof of part (a). Describe how you would modify the proof of part (a) to provide proofs of part (b) and part (c).