

Lecture #9: Nonregular Languages, Part Two

Lecture Presentation

Main Points

Closure Properties for Regular Languages That We Know About

Why These Can Be Used to Prove that Languages are *not* Regular

First Problem

Let $\Sigma = \{a, b\}$. We wish to prove that the language

$$L = \{\omega \in \Sigma^* \mid \text{the number of a's in } \omega \text{ is equal to the number of b's in } \omega\} \subseteq \Sigma^*$$

is not a regular language.

A Similar Language That We Know is Not Regular

How are These Languages Related?

Using This to Write a Proof

Second Problem

We now wish to prove the following.

Claim: For every alphabet Σ and for languages $L_1, L_2 \subseteq \Sigma^*$, if L_1 is a regular language and L_2 is a regular language then their **intersection**, $L_1 \cap L_2$, is also a regular language.

Can $L_1 \cap L_2$ Be Obtained Using Other Operations on Sets?

Hint: Consider “De Morgan’s laws”.

How Can This Be Used To Prove the Claim?

Third Problem

How can the new closure property (for the intersection of languages), that has now been established, be used to find a *different* solution for the first problem?

Conclusions

What can you reasonably conclude about closure properties, and their use to prove that languages are not regular, after considering these problems?