Lecture #7: Regular Expressions and Regular Operations
What Will Happen During the Lecture

Review

The lecture presentation will begin with a ***brief*** review of the material in the preparatory video and documents for this lecture — and students will have the chance to ask questions about this.

Recognition and Application of a Regular Expression

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ —- so that that does not include any of the symbols

$$\lambda, \emptyset, \Sigma, (,), \cup, \circ, ^\star$$

and let

$$\Sigma_{\text{regexp}} = \Sigma \cup \{\lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\}.$$

Given a string $\omega \in \Sigma^\star_{\text{regexp}}$, you might wish to do each of the following things:

- Decide whether ω is a regular expression over Σ.
- If ω *is* a regular expression over Σ, then — for a given string $\mu \in \Sigma^\star$ — decide whether μ belongs to the language of ω.
- If ω *is* a regular expression over Σ, then *describe* the language of ω.

These problems will be discussed using the string

$$\omega = ((((\Sigma)^\star \circ \mathtt{a}) \circ (\Sigma)^\star) \circ (\mathtt{a} \circ (\Sigma)^\star)) \in \Sigma^\star_{\text{regexp}}$$

and (for the second part of the problem) the strings $\mu_1 = \mathtt{abaca} \in \Sigma^\star$ and $\mu_2 = \mathtt{bac} \in \Sigma^\star$.

Designing a Regular Expression

If someone describes a language $L \subseteq \Sigma^\star$ — and L is a regular language — how can you discover a regular expression ω (over the alphabet Σ) whose language is L?

During the lecture presentation this question will be considered — as we design a regular expression — over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, once again — for the language consisting of all strings in $\Sigma^\star$ that include an even number of copies of the symbol "a" — that is, for the language

$$L = \{\mu \in \Sigma^\star \mid \text{the number of copies of “a” in } \mu \text{ is divisible by } 2\}.$$