Lecture #7: Regular Expressions and Regular Operations

Lecture Presentation

Main Points

Recognition and Application of a Regular Expression

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ —- so that that does not include any of the symbols

$$\lambda, \emptyset, \Sigma, (,), \cup, \circ, ^\star$$

and let

$$\Sigma_{\text{regexp}} = \Sigma \cup \{\lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \lambda, \emptyset, \text{“}\Sigma\text{”}, (,), \cup, \circ, ^\star\}.$$

Consider the string

$$\omega = ((((\Sigma)^\star \circ \mathtt{a}) \circ (\Sigma)^\star) \circ (\mathtt{a} \circ (\Sigma)^\star)) \in \Sigma_{\text{regexp}}^\star.$$

It turns out ω *is* a regular expression over $\Sigma^\star$. ***How could you prove this?***

Consider the string $\mu_1 = \texttt{abaca} \in \Sigma^\star$. It turns out that μ_1 is in the language of the regular expression ω. ***How could you prove this?***

Consider the string $\mu_2 = \texttt{bac} \in \Sigma^\star$. It turns out that μ_2 is *not* in the language of the regular expression ω. ***How could you prove this?***

What is the *language* of ω? ***How could you prove this?***

What is another way to prove that μ_1 belongs to the language of ω, but ω_2 is not in the language of ω, using the information that we have now?

Designing a Regular Expression for a Given Language

Once again, let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and let L be the set of all strings in $\Sigma^\star$ that include an even number of copies of the symbol "a" — that is, the language

$$L = \{\mu \in \Sigma^\star \mid \text{the number of copies of "a" in } \mu \text{ is divisible by } 2\}.$$

This a regular language; suppose that we want to design a regular expression, over Σ, whose language is L.

Strategy for Discovery of a Regular Expression

A Simpler Language

An Even Simper Language

Working Our Way Back Up to L